

Dare spazio ai numeri: l'apprendimento delle prime abilità numeriche alla scuola dell'infanzia

Francesco Sella

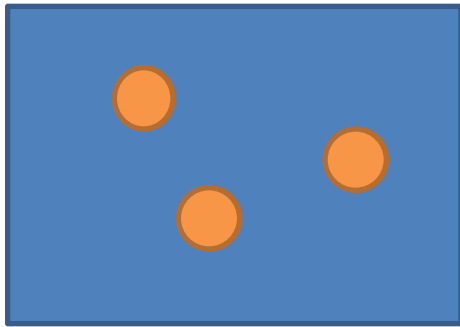
Department of Experimental Psychology
University of Oxford

francesco.sella@psy.ox.ac.uk



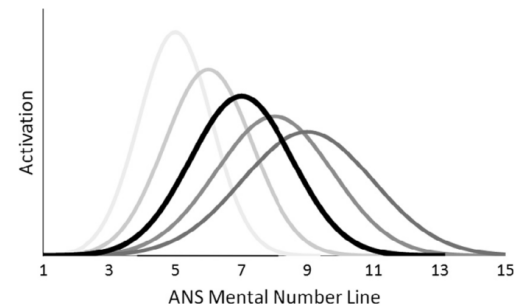
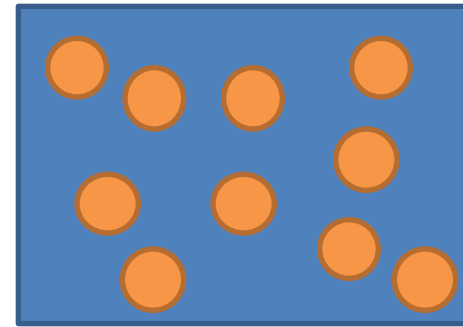
Neuro-cognitive start-up tools

Object Tracking System (OTS)



- Individuazione rapida ed accurata di un numero limitato di elementi (3-4).
- Subitizing
- VSTM

Approximate Number System (ANS)

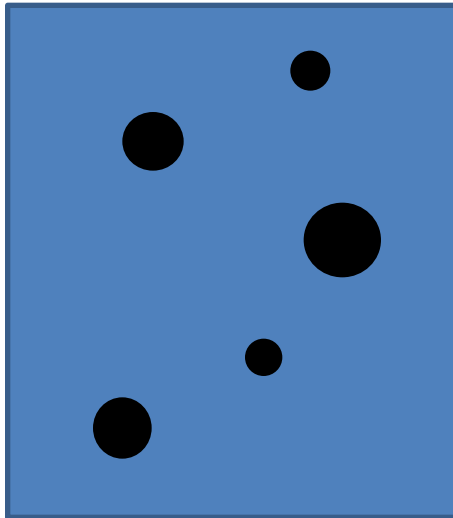


Dal non-simbolico al simbolico

Symbol-grounding

Natura

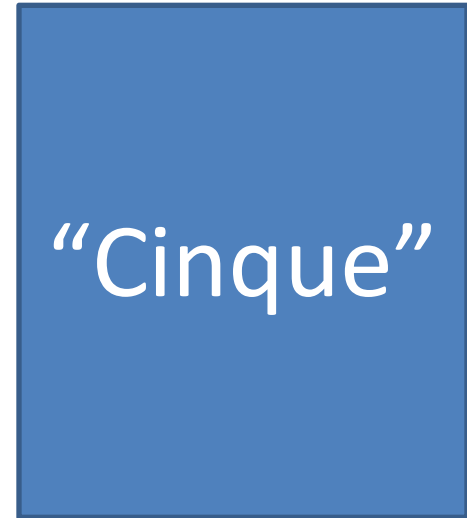
Non-simbolico



Cultura

Simbolico

“Cinque”

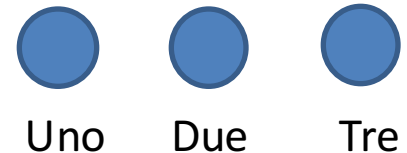


Conteggio

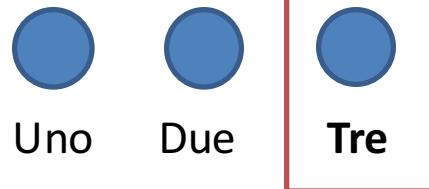
Principi del conteggio (Gelman & Gallistel, 1978):

- **Ordine stabile:** «uno, due, tre, quattro...»

- **Corrispondenza biunivoca (1-1)**



- **Cardinalità**



Knower-level

Give-a-number → Stadi di sviluppo del principio di cardinalità (Sarnecka & Carey, 2008; Wynn, 1990):

- Pre-number-knower
- One-knower
- Two-knower
- Three-knower
- **Four-knower**
- **Cardinal-principle knower**

Subset-knowers



Cinque pomodori,
grazie!



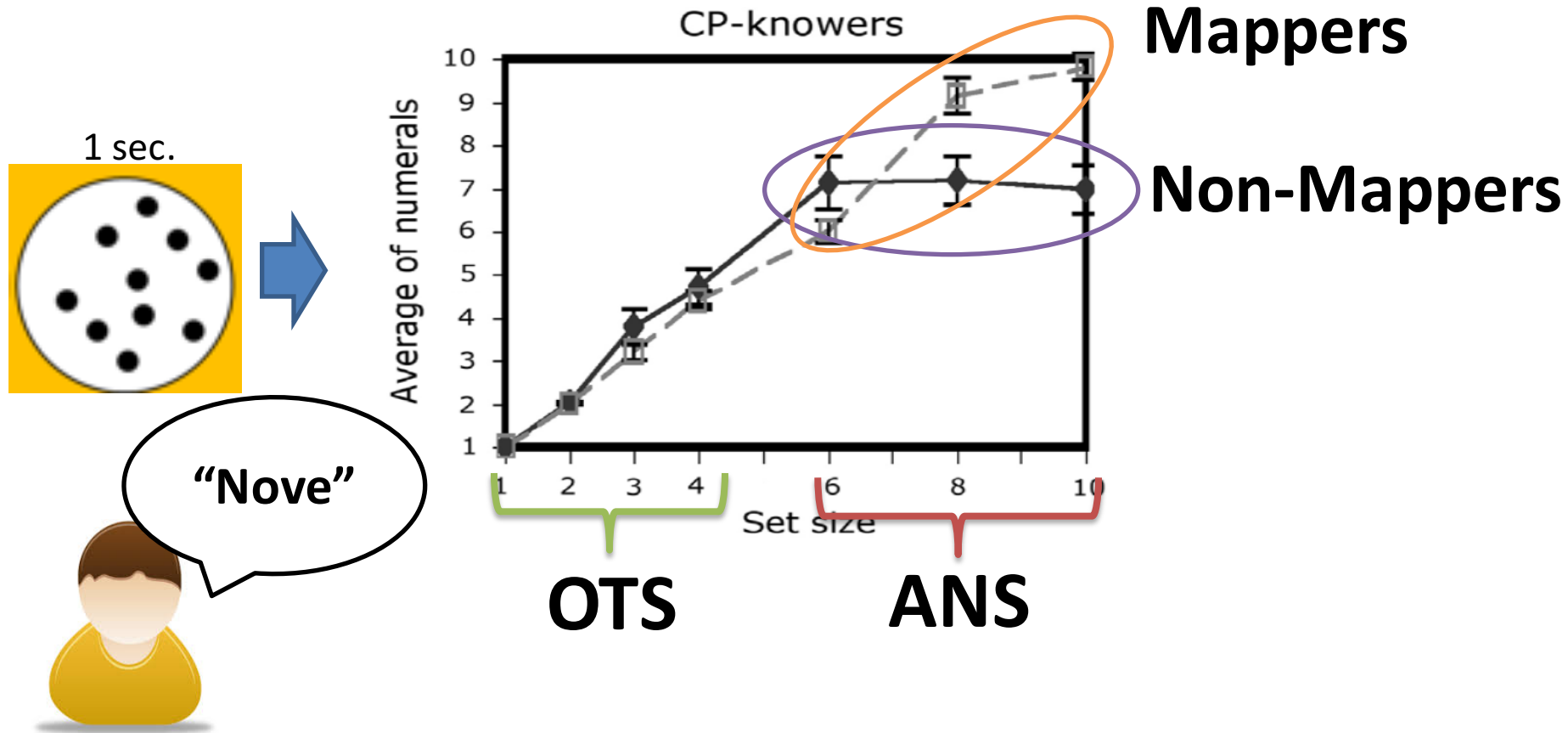
OTS +

Conceptual bootstrapping (Carey, 2004)

Il principio di cardinalità implica un *mapping* fra lista numerica e numerosità?

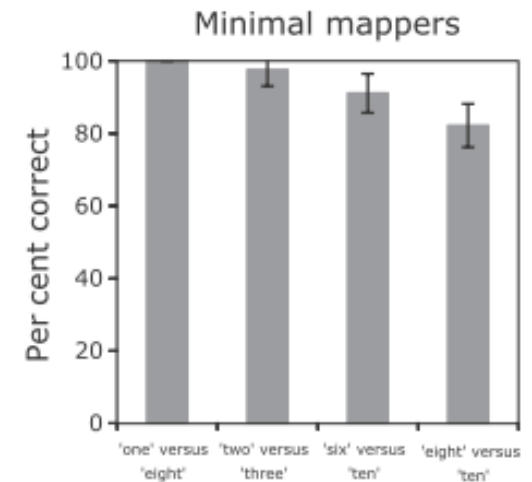
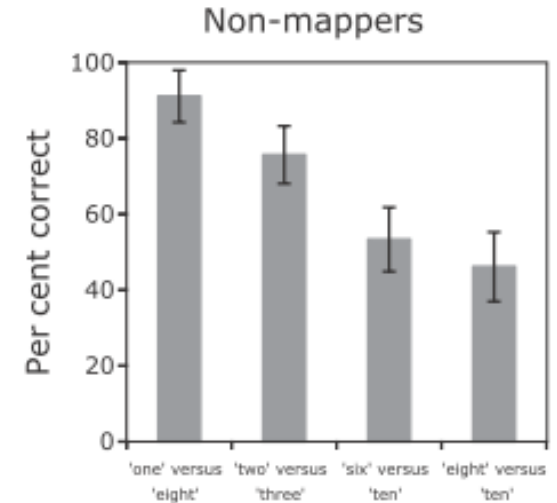
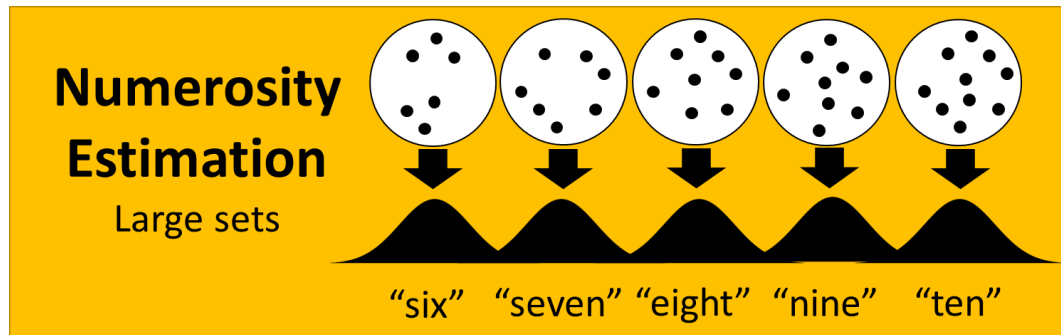
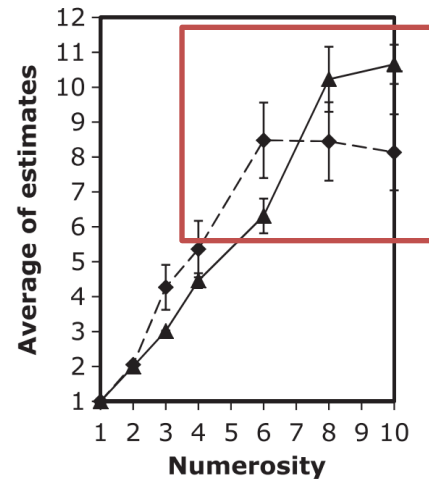
CP-mappers e CP-non-mappers

Stima di numerosità → Le Corre et al. (2007; 2014; Odic et al., 2015).



CP-mappers e CP-non-mappers

Le Corre et al. (2014)



Conteggio e mapping spaziale

Exp1: 46 bambini prescolari (29 Maschi; Etá in mesi= 61, SD = 6, range = 48-70).

Exp2: 46 bambini prescolari (28 Maschi; Etá in mesi= 58, SD = 11, range = 41-77).

- Give a number → Subset-knowers e CP-knowers
- Linea numerica

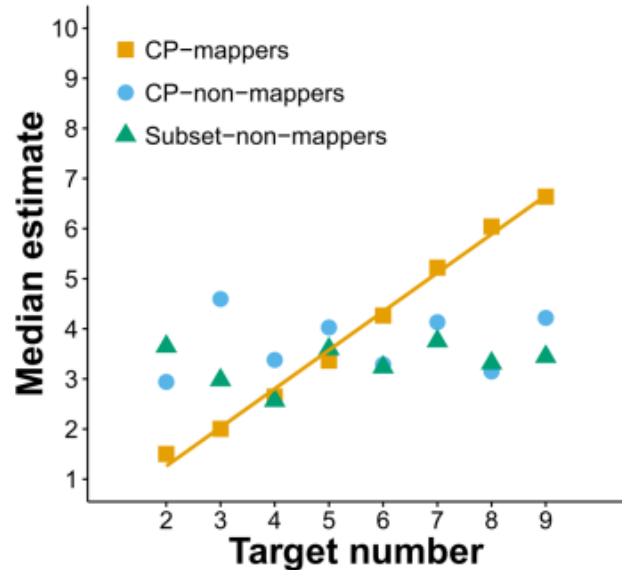
7



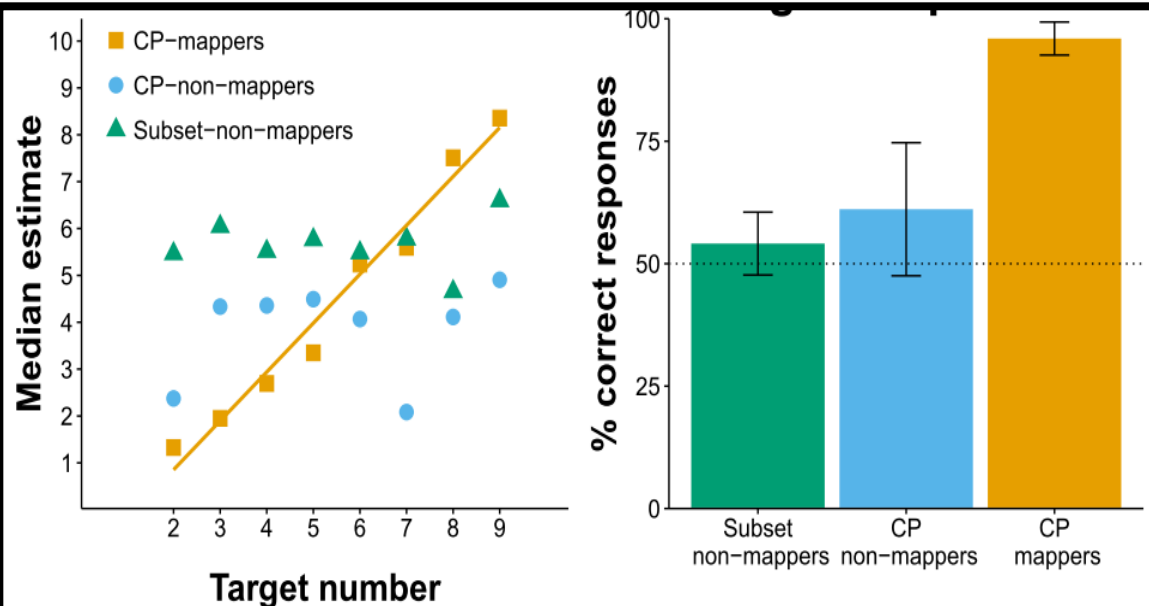
- Comparazione numeri arabi: 9 vs 6

Conteggio e mapping spaziale

Exp 1



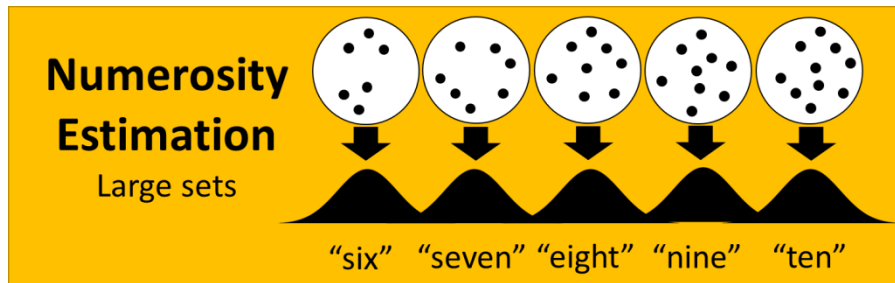
Exp 2



Spatial mapping principle

Mapping Verbale: ANS-lista numerica

Le Corre et al. 2007; 2014.



Comparazione verbale

“otto” vs. “Dieci”

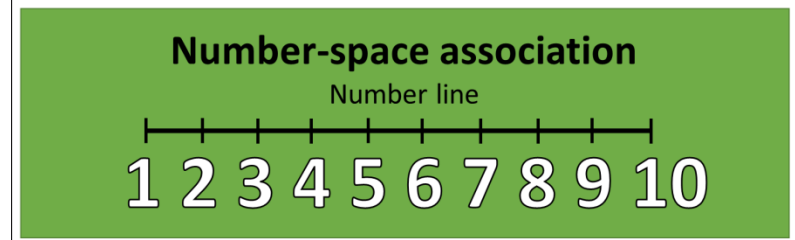
Mapping spaziale: Numeri e spazio

Sella et al. (2017)

Sella, Lucangeli, Zorzi (under review)

Sella, Lucangeli, Zorzi (under review)

Spatial mapping



Comparazione visiva

8 vs. 10

Conteggio e mapping spaziale dei numeri

53 prescolari (21 maschi; Eta in mesi = 63, SD = 10, range = 43-79).

Tasks:

- **GaN.**
- **NL 1-10.**
- **Stima numerica (as Le Corre 2014).**
- **Comparazione verbale (as Le Corre 2014).**
- **Comparazione visiva.**

Risultati

Comparazione verbale (es., «otto» vs. «quattro»)

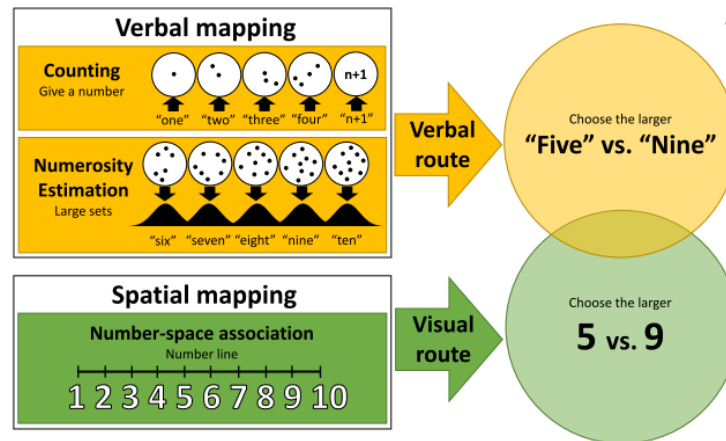
Modello	Misure	Comparazione	BF_{10m2}/BF_{10m1}
1	GaN (% corrette)	Null vs 1	144
2	GaN (% corrette)	1 vs 2	0.74
	Linea numerica (PAE)		
3	GaN (% corrette)	1 vs 3	1.11
	Stima di numerosità (differenza assoluta >4)		

Comparazione visiva (es., 8 vs. 4)

Modello	Misure	Comparazione	$BF_{m2/m1}$
1	GaN (% corrette)	Null vs 1	395
2	GaN (% corrette)	1 vs 2	36
	Linea numerica (PAE)		
3	GaN (% corrette)	1 vs 3	1.71
	Stima di numerosità (differenza assoluta >4)		
4	Lettura numeri (% corrette)	Null vs 4	>1000
	Comparazione verbale (% corrette)		
5	Lettura numeri (% corrette)	4 vs 5	5.69
	Comparazione verbale (% corrette)		
	Linea numerica (PAE)		

Conclusioni in itinere

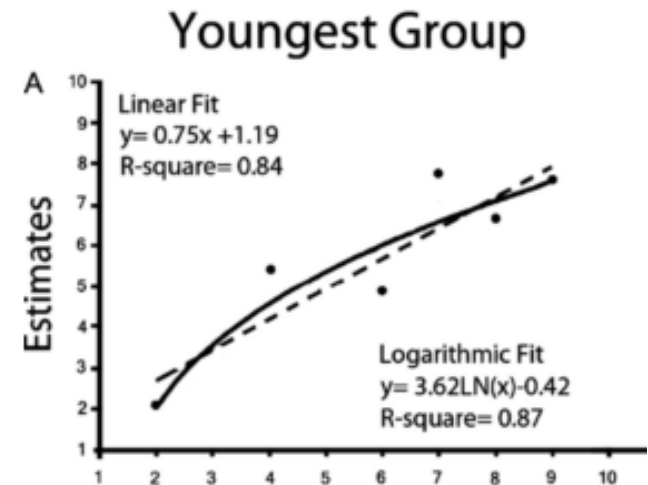
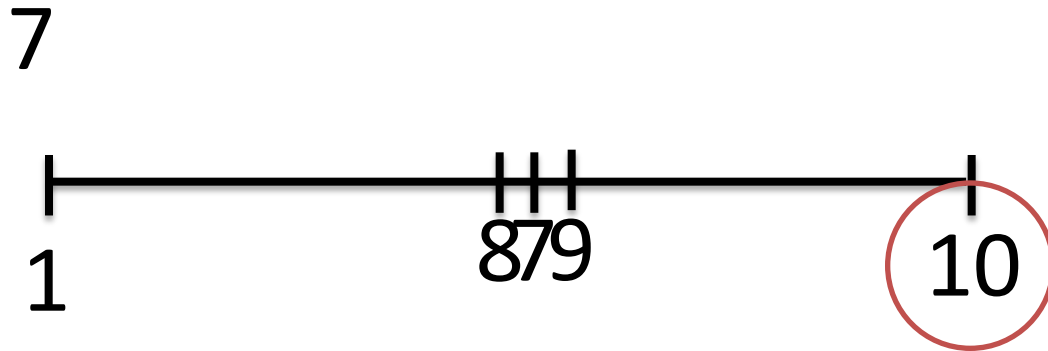
- Principio di cardinalità (GaN) → risposta alla domanda « quanti sono? »
- Esatta rappresentazione simbolica: comparazione numeri arabi (Schneider et al., 2016).
- La capacità di comparare numeri dipende dal formato (verbale o visivo) di presentazione.



- Spazio e grandezza numerica → Correlazione vs. causalità (vedere Siegler & Ramani, 2009).

Spatial mapping: Direzione, ordine o spazio?

Linea numerica



Berteletti et al. (2010)

- Direzione fissa S-D.
- Intervallo fisso (1-10).
- Mapping compresso: Ordine? Spazio?

Quale componente predice la comparazione di numeri?

Participants and procedure

62 bambini prescolari (31 Maschi; Eta in mesi= 69, SD = 4).

Compiti

- **GaN: tutti CP-knowers.**
- **Enumerazione in Avanti (max 50).**
- **Stima di numerosità (da 4 a 10).**
- **Comparazione di triplette di numeri**
- **DOS task**

Digit comparison task

7 triplette per 6 possible permutazioni = 42 trials.

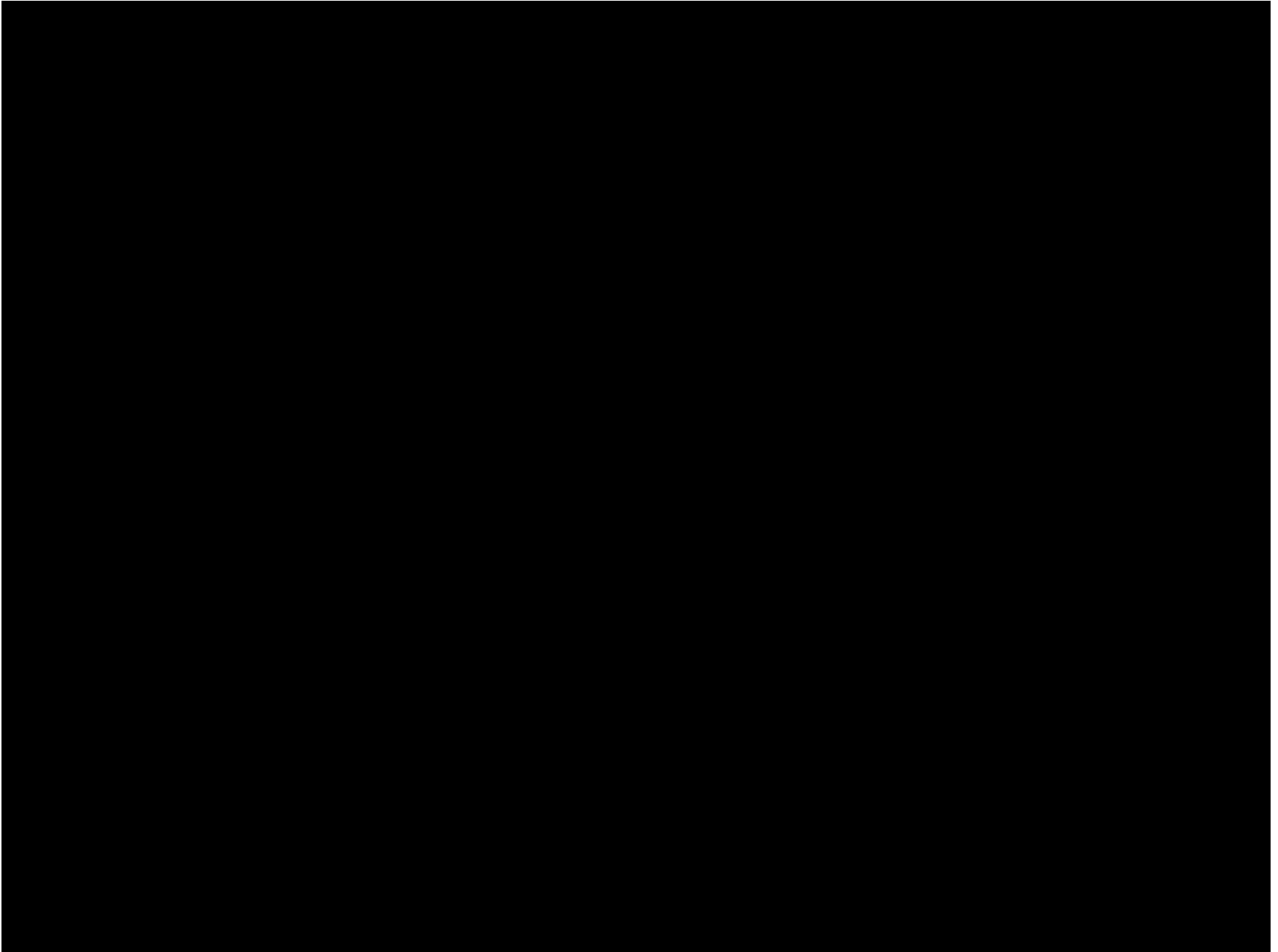
Accuratezza: deviazione dalla risposta corretta.

Tutti i numeri erano letti dallo sperimentatore.



2 3 1

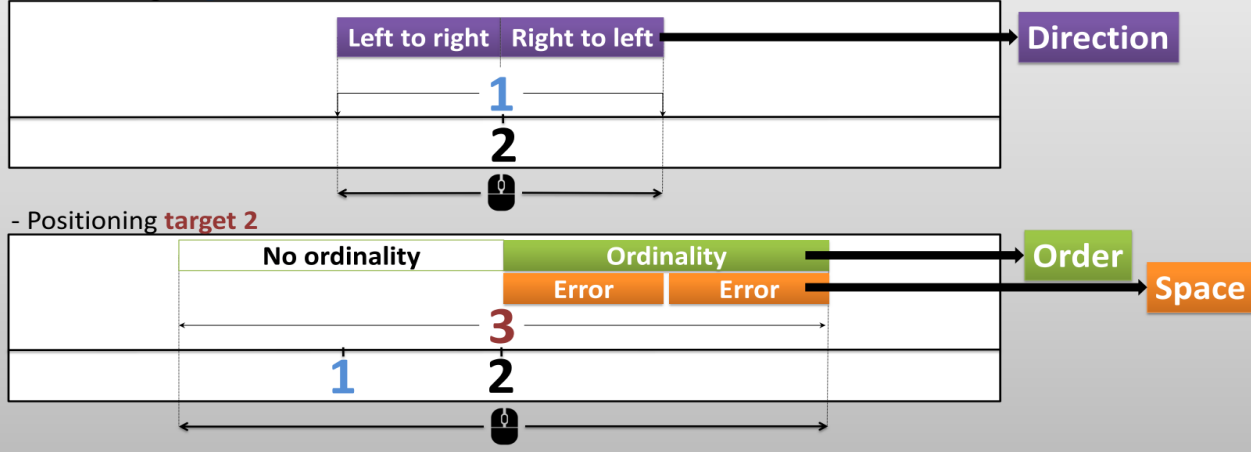
Direction-Order-Space task



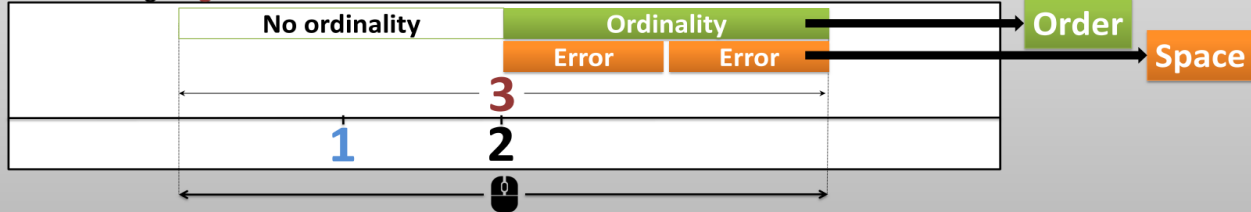
Direction-Order-Space task

a) Two-side trial

- Positioning **target 1**

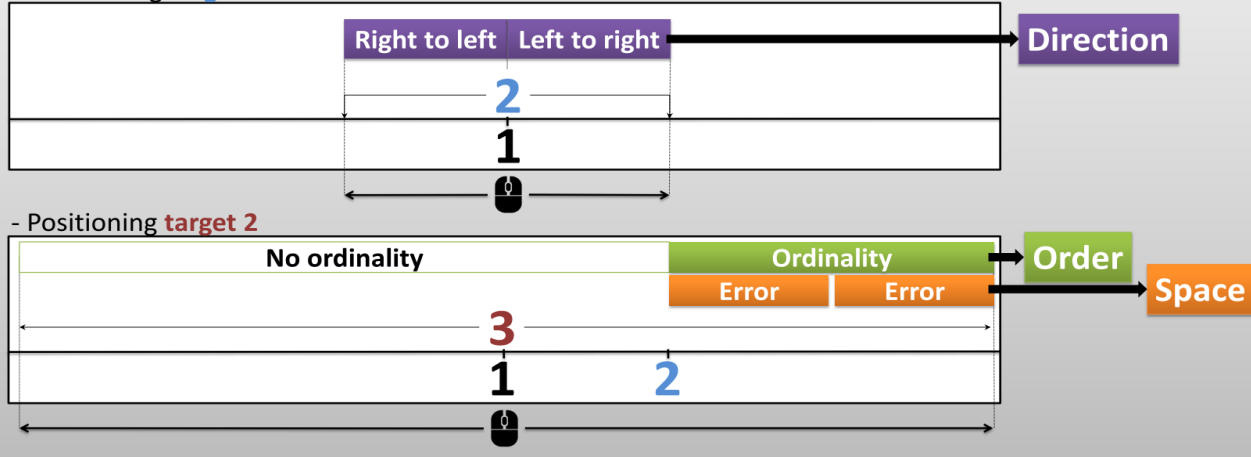


- Positioning **target 2**

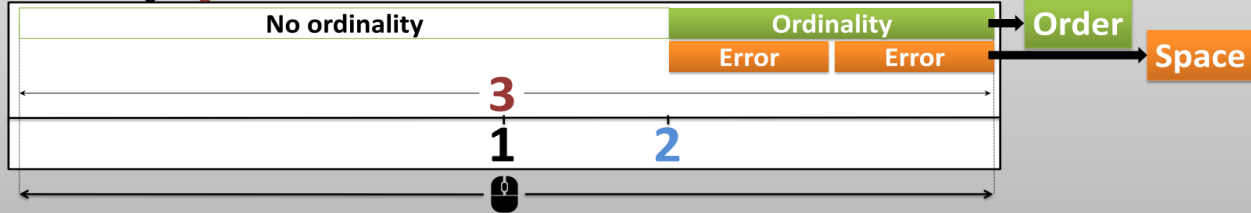


b) One-side trial

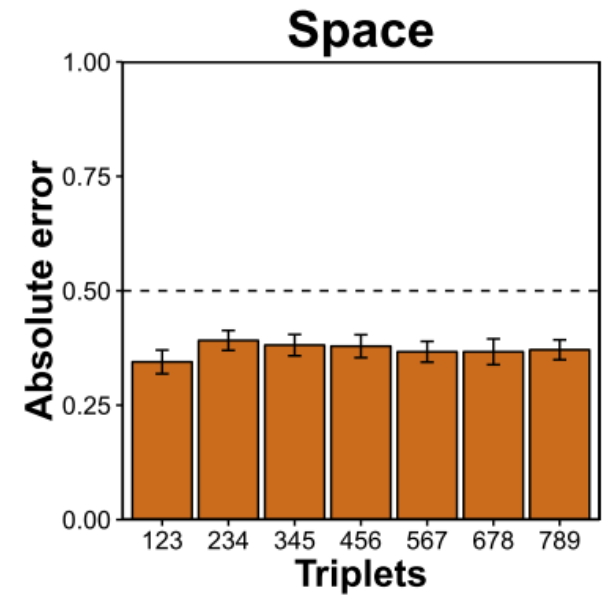
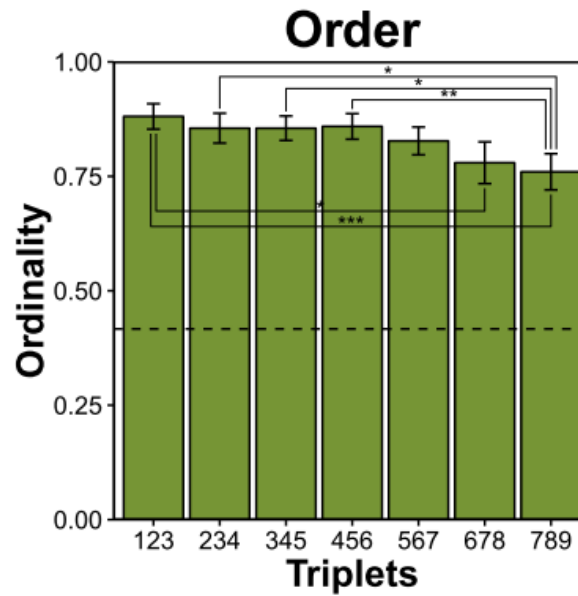
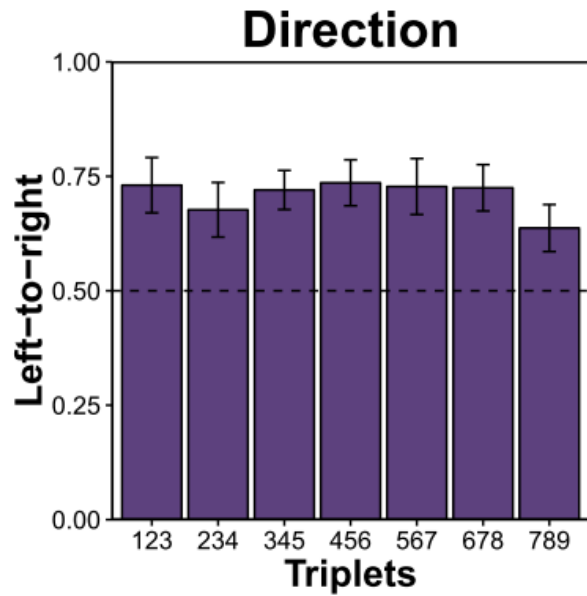
- Positioning **target 1**



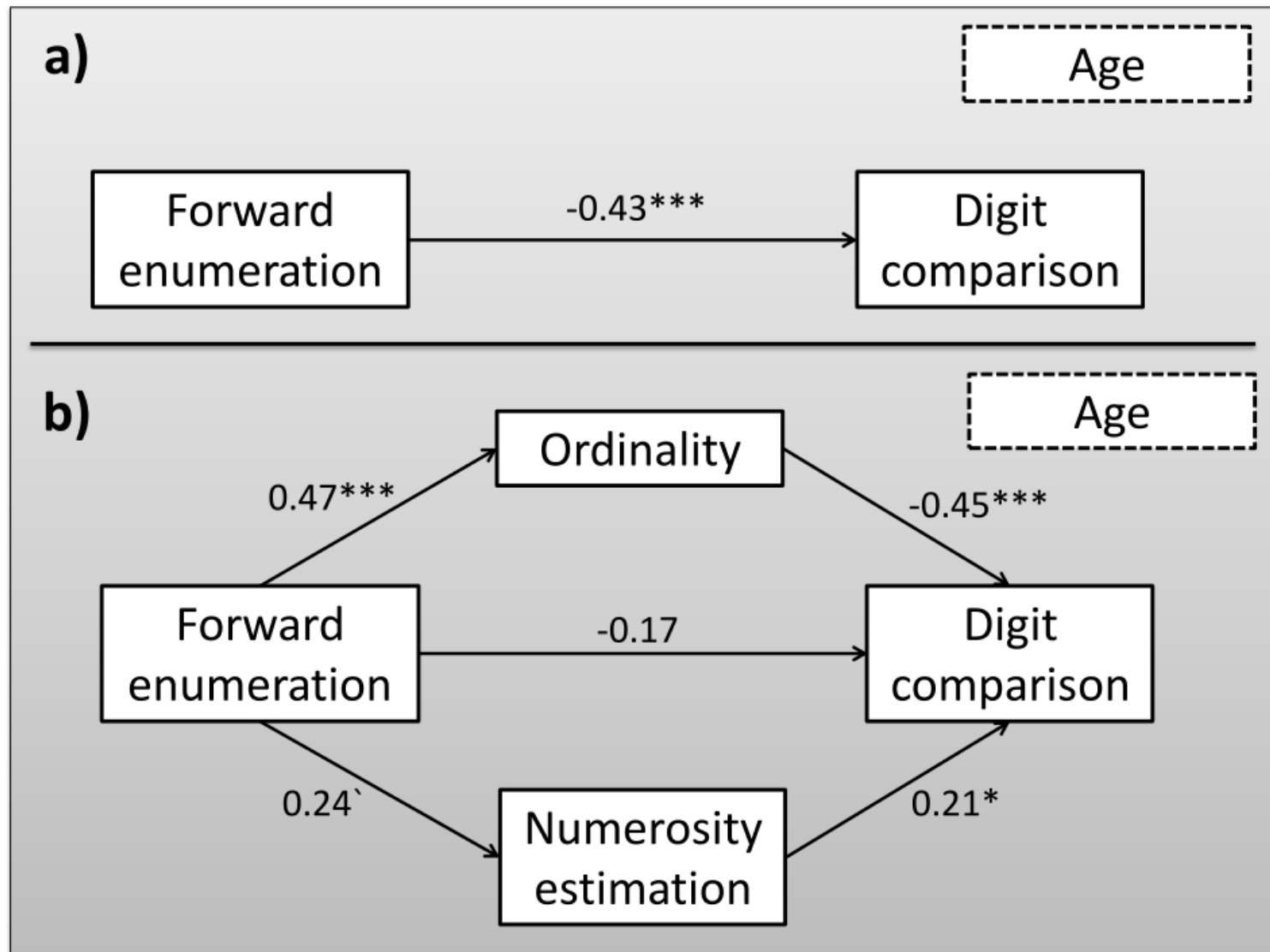
- Positioning **target 2**



DOS task

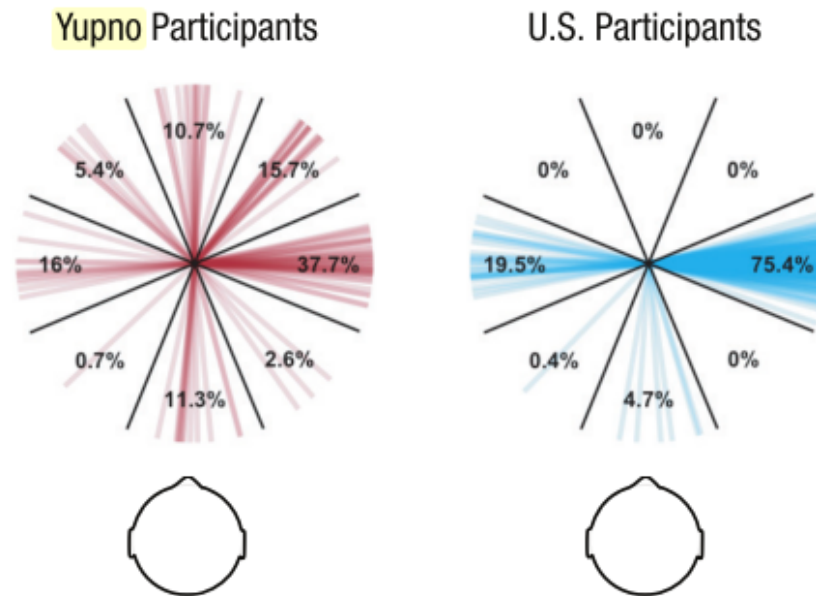


Mediazione



Conclusioni

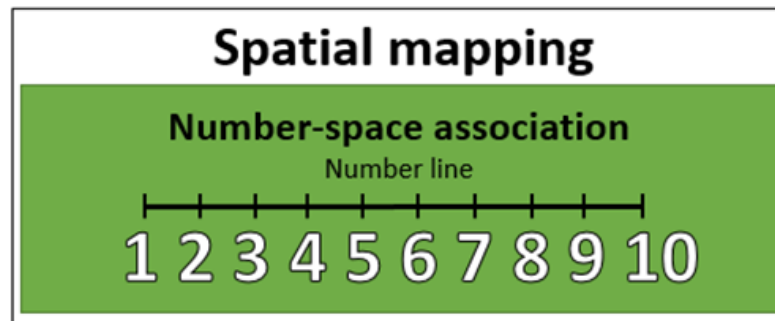
- Mapping principalmente da **sinistra a destra** ma con un certo grado di flessibilità (McCrink & Opfer, 2014; Opfer, Thompson, & Furlong, 2010; Patro, Fischer, Nuerk, & Cress, 2016) → **Infuenza della direzione del conteggio?** (Rinaldi et al., 2016)



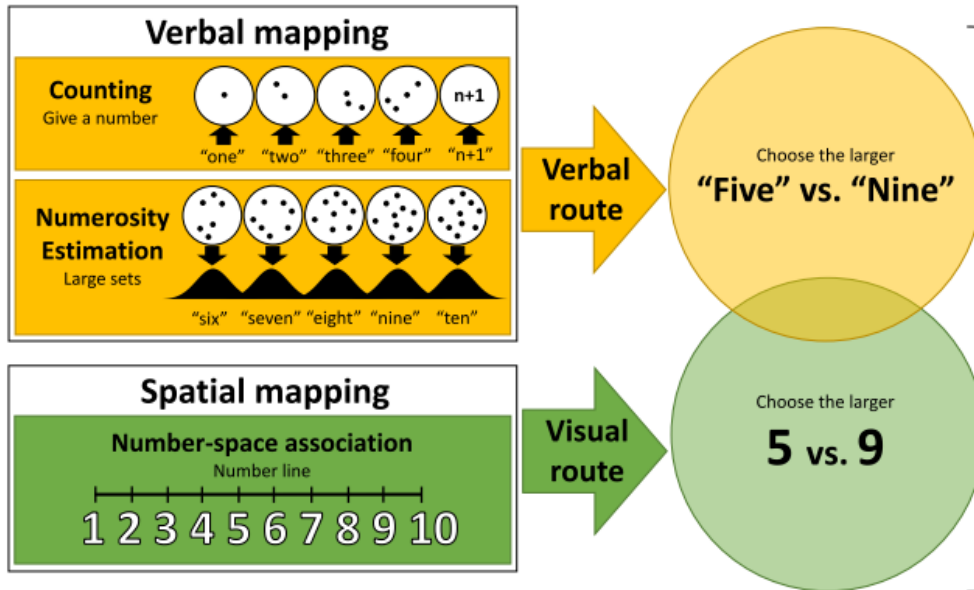
Cooperrider, Marghetis, and Núñez (2017)

Conclusioni

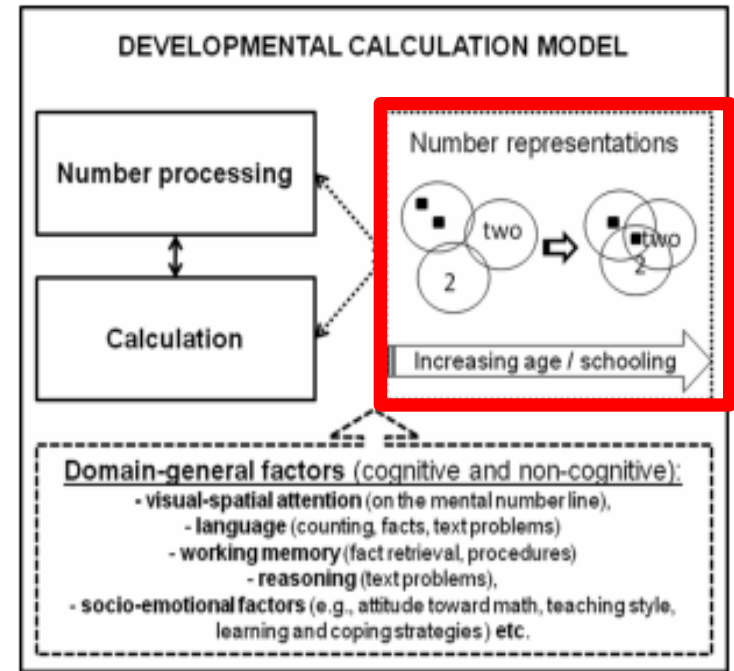
- Stretta relazione fra **ordine spaziale** e **comprensione della grandezza numerica**.
- La disposizione spaziale dei numeri come schema di riferimento in cui un **numero assume grandezza numerica in base alla sua posizione** ed in relazione con la posizione degli altri numeri (symbol-symbol association; Reynvoet & Sasanguie, 2016).
- Comprensione della **struttura della linea numerica**.



Conclusione generale

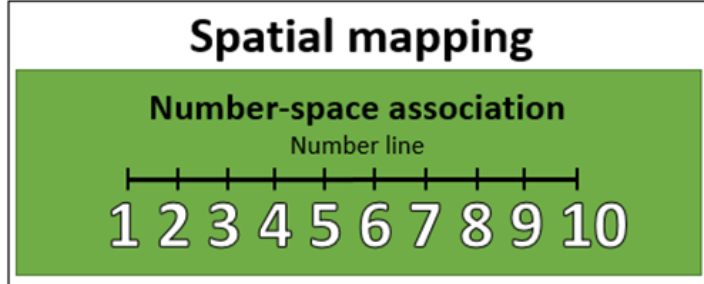


See Benoit et al., 2013; Hurst, Anderson, & Cordes, 2016; Knudsen et al., 2015; Jimenez-lira et al., 2017.



(Kaufmann et al., 2011)

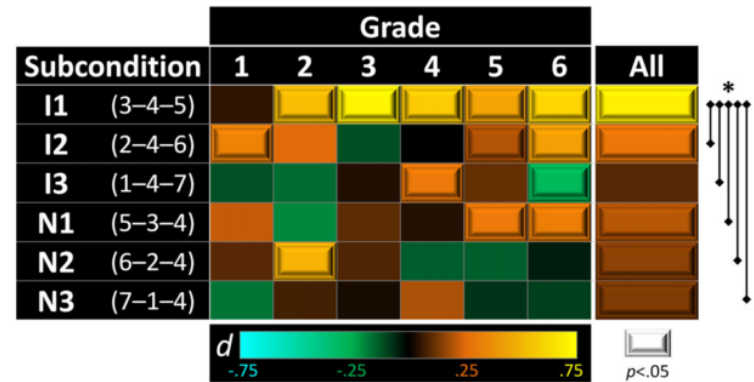
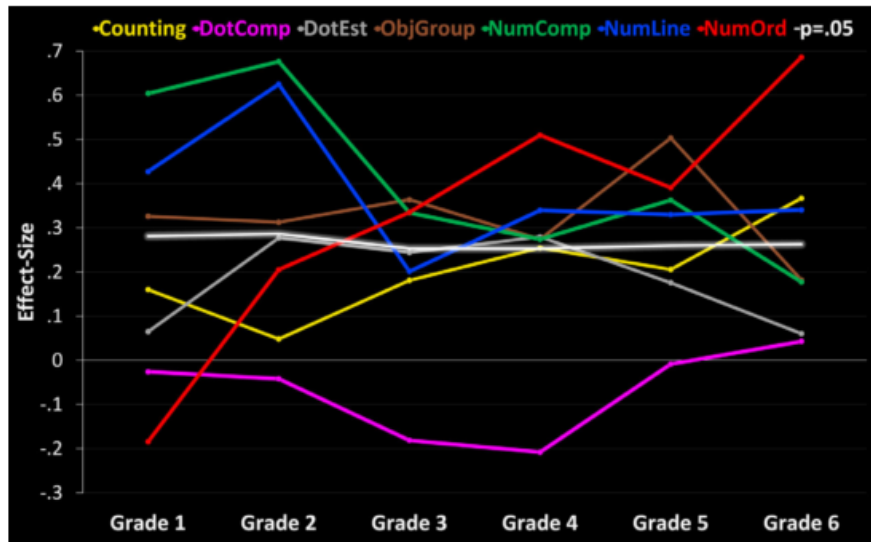
Conclusioni



Ordine spaziale o verbale?



Conoscenza ed accesso alla linea numerica.



(Lyons et al. 2014; Lyons & Ansari, 2015)



Ilaria Berteletti



Daniela Lucangeli

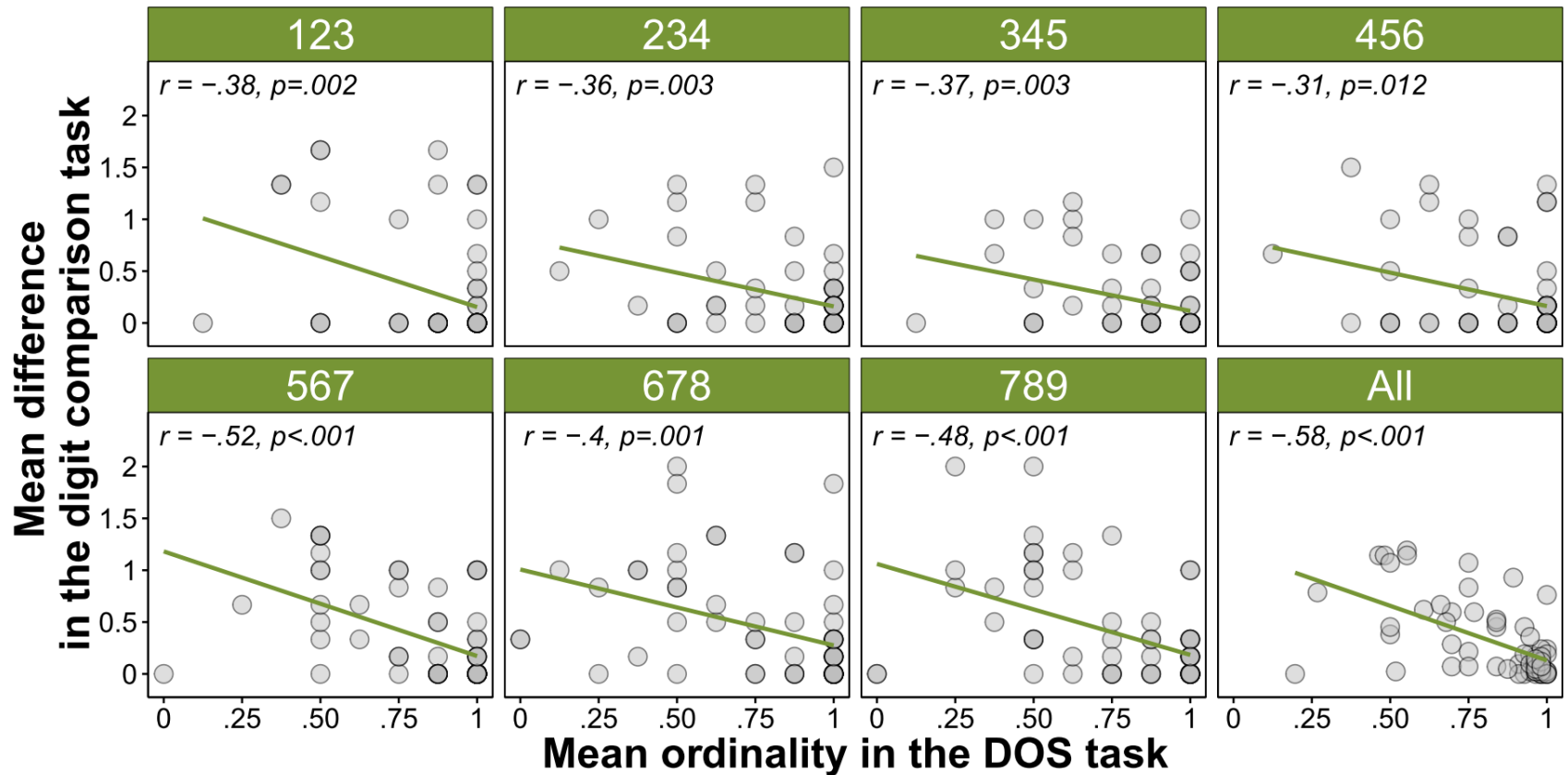


Marco Zorzi

Grazie a tutti i bambini, gli insegnanti e i genitori che hanno preso parte ai nostri studi ed alle studentesse che hanno collaborato alla raccolta dati (Sara Borsatto, Anna Cappellari, Alice Pasqualotto, Roberta Zamprogno, Margherita Trevisiol).

Grazie per l'attenzione!
sella.francesco@gmail.com

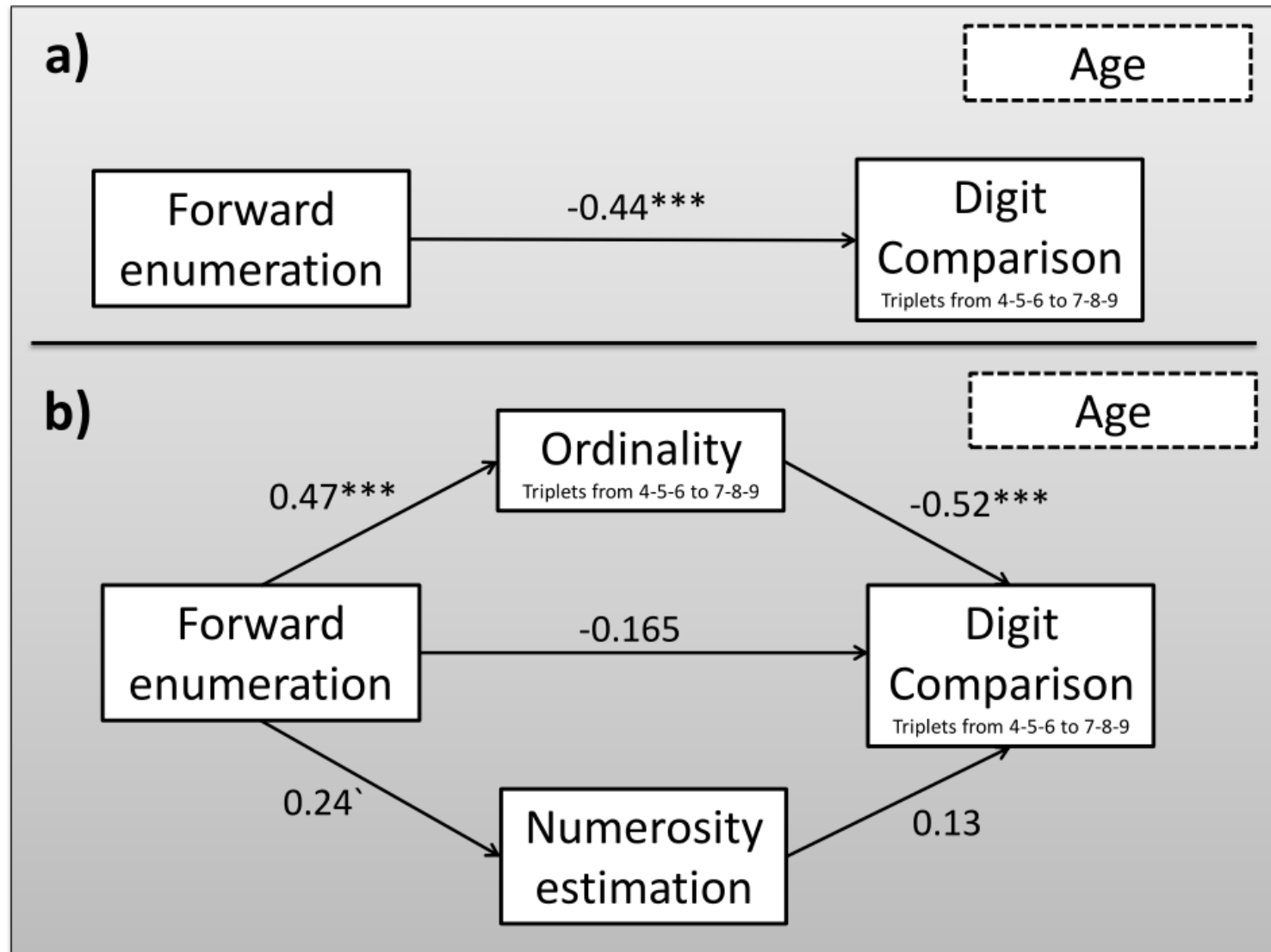
Digit comparison and Ordinality



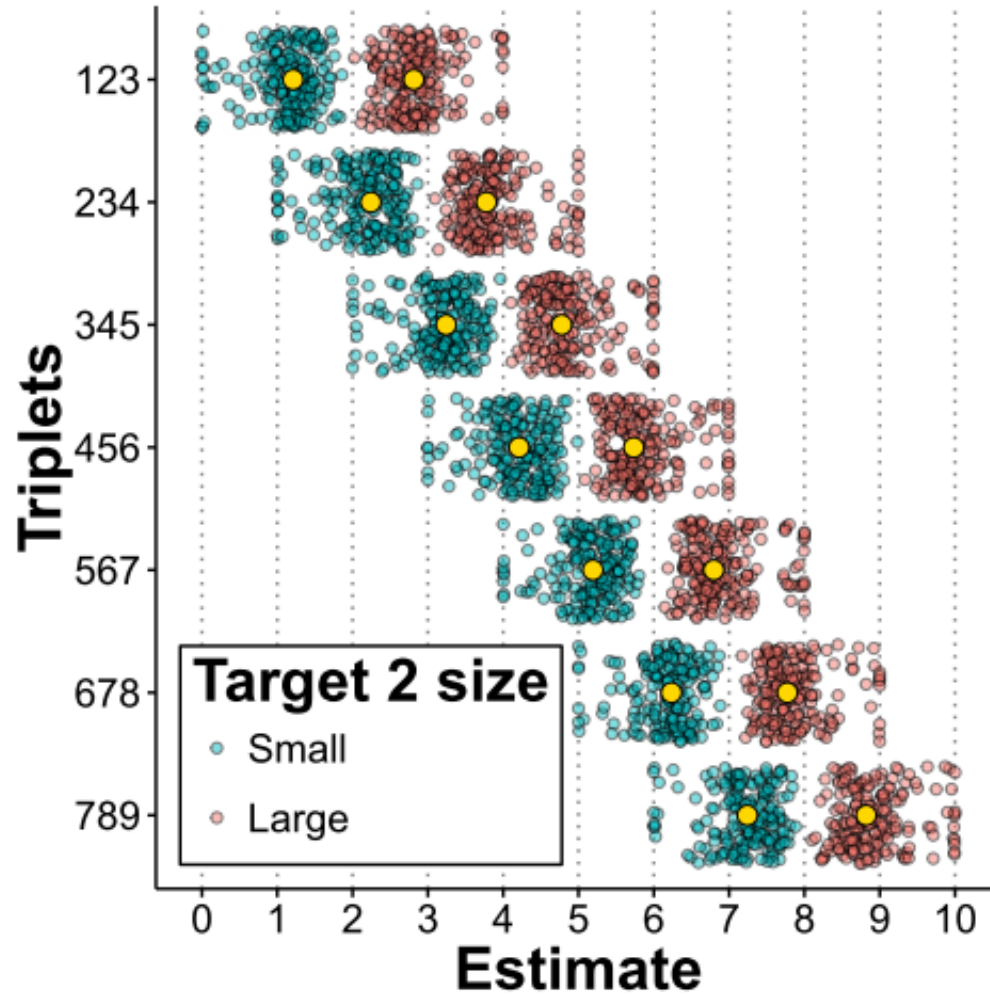
Regression models

Model	Measures	B	95% CI		Model comparisons	ΔR^2
1	Age	-0.01	-0.03	0.01	Null vs. 1	.01
2	Age	-0.02	-0.035	0.005	2 vs. 1	.35***
	DOS ordinality	-1.08***	-1.46	-0.7		
3	Age	-0.01	-0.04	0.01	3 vs. 1	.03
	DOS direction	-0.21	-0.54	0.12		
4	Age	-0.01	-0.03	0.01	4 vs. 1	.05
	DOS space	0.60	-0.08	1.28		
5	Age	-0.005	-0.03	0.02	5 vs. 1	.15**
	Numerosity estimation	0.18**	0.07	0.28		
6	Age	-0.01	-0.03	0.01	6 vs. 2	.05*
	DOS ordinality	-0.95***	-1.34	-0.56	6 vs. 5	.24***
	Numerosity estimation	0.10*	0.01	0.20		
7	Age	-0.01	-0.03	0.005	7 vs. 2	.02
	DOS ordinality	-1.08***	-1.49	-0.67		
	DOS direction	0.08	-0.21	0.36		
	DOS space	0.40	-0.17	0.97		
8	Age	-0.02	-0.04	0.005		
	DOS ordinality	-1.08***	-1.49	-0.67		
	DOS direction	0.07	-0.22	0.36		
	DOS space (residuals)	0.35	-0.22	0.92		

Mediation model: large numbers

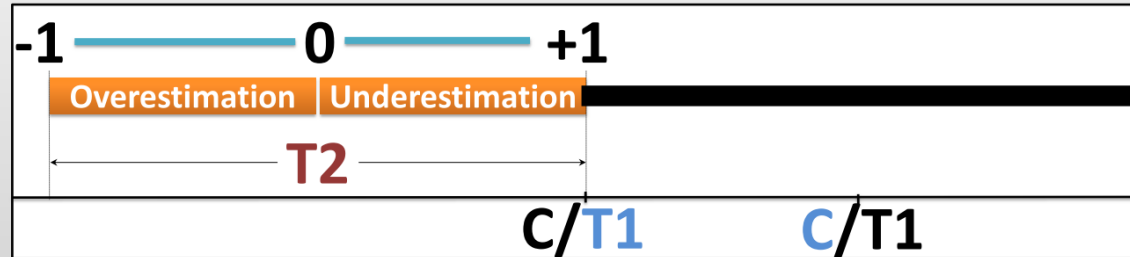


Spatial mapping of digits

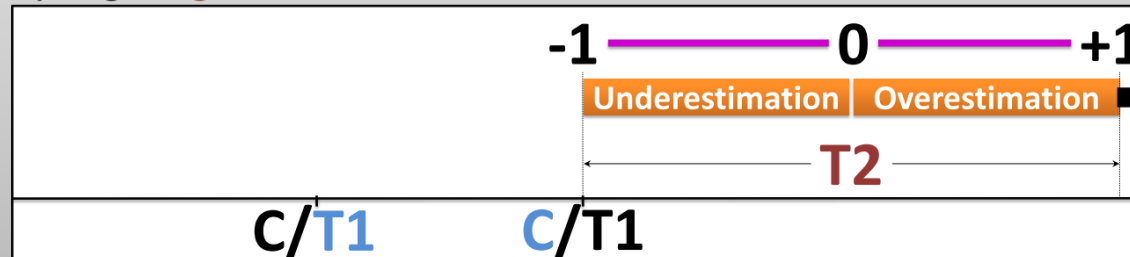


Spatial mapping of digits

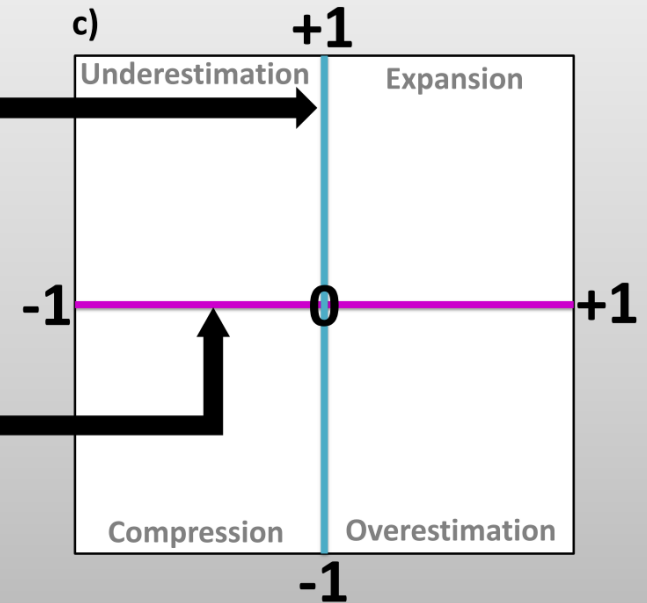
a) Small target 2



b) Large target 2

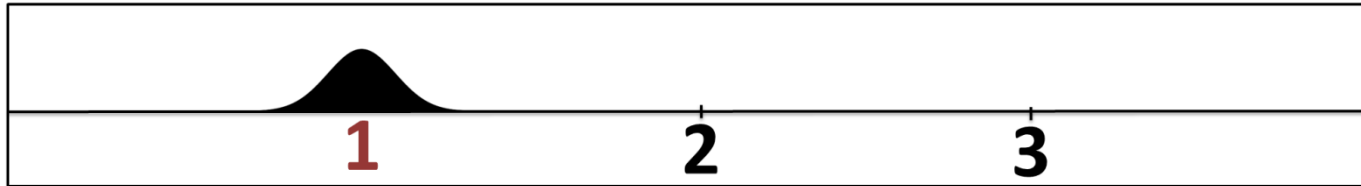


c)

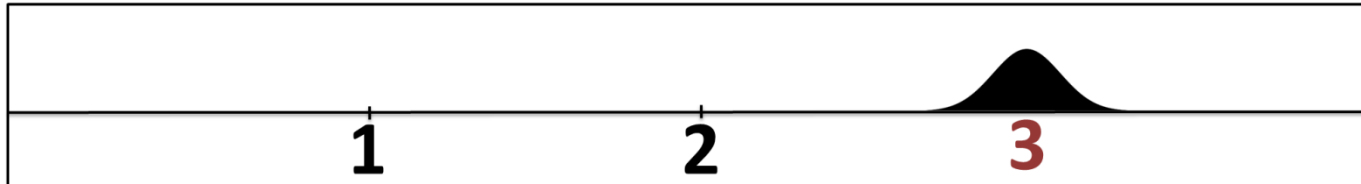


Spatial mapping of digits

- Accurate estimation of small **target 2**



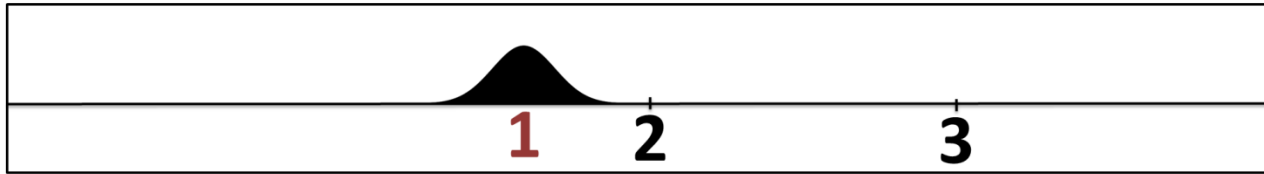
- Accurate estimation of large **target 2**



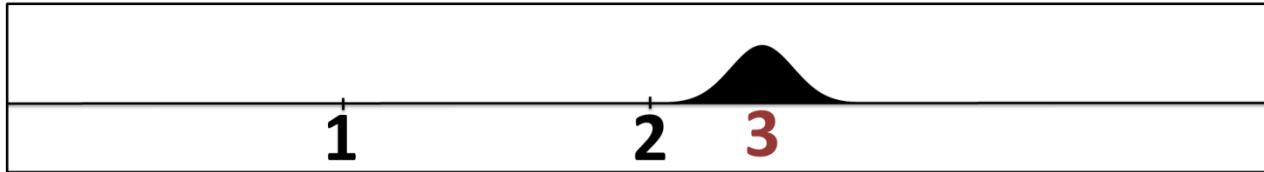
Linear
mapping

Spatial mapping of digits

- Underestimation of small **target 2**



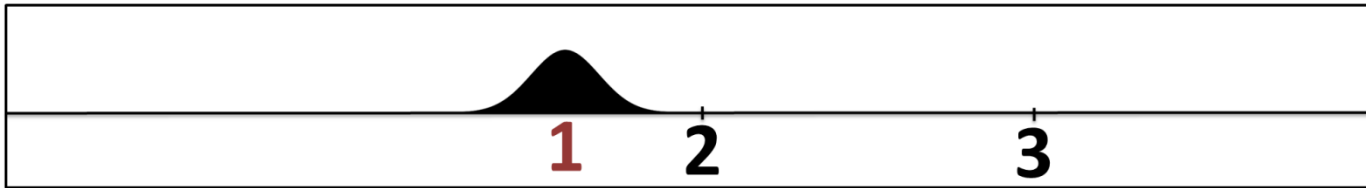
- Underestimation of large **target 2**



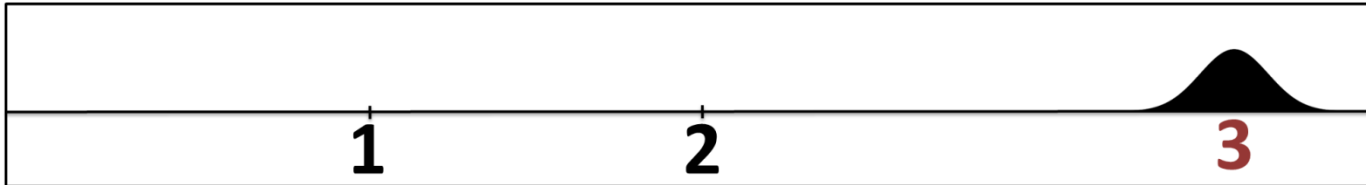
Underestimation
mapping

Spatial mapping of digits

- Underestimation of small **target 2**



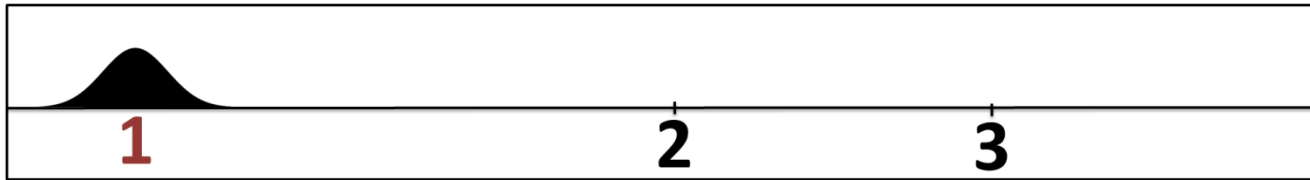
- Overestimation of large **target 3**



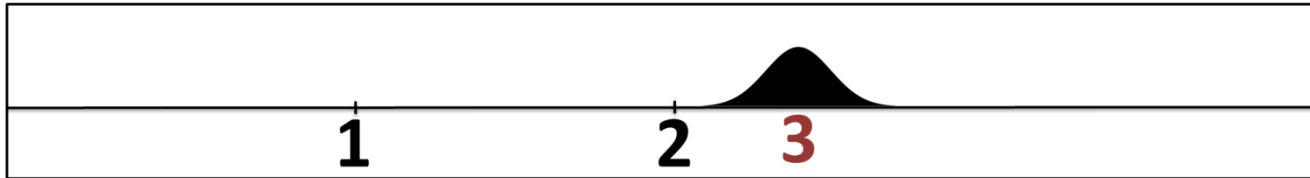
Expansion
mapping

Spatial mapping of digits

- Overestimation of small **target 2**



- Underestimation of large **target 3**



Compressed
mapping

Spatial mapping of digits

- Overestimation of small **target 2**

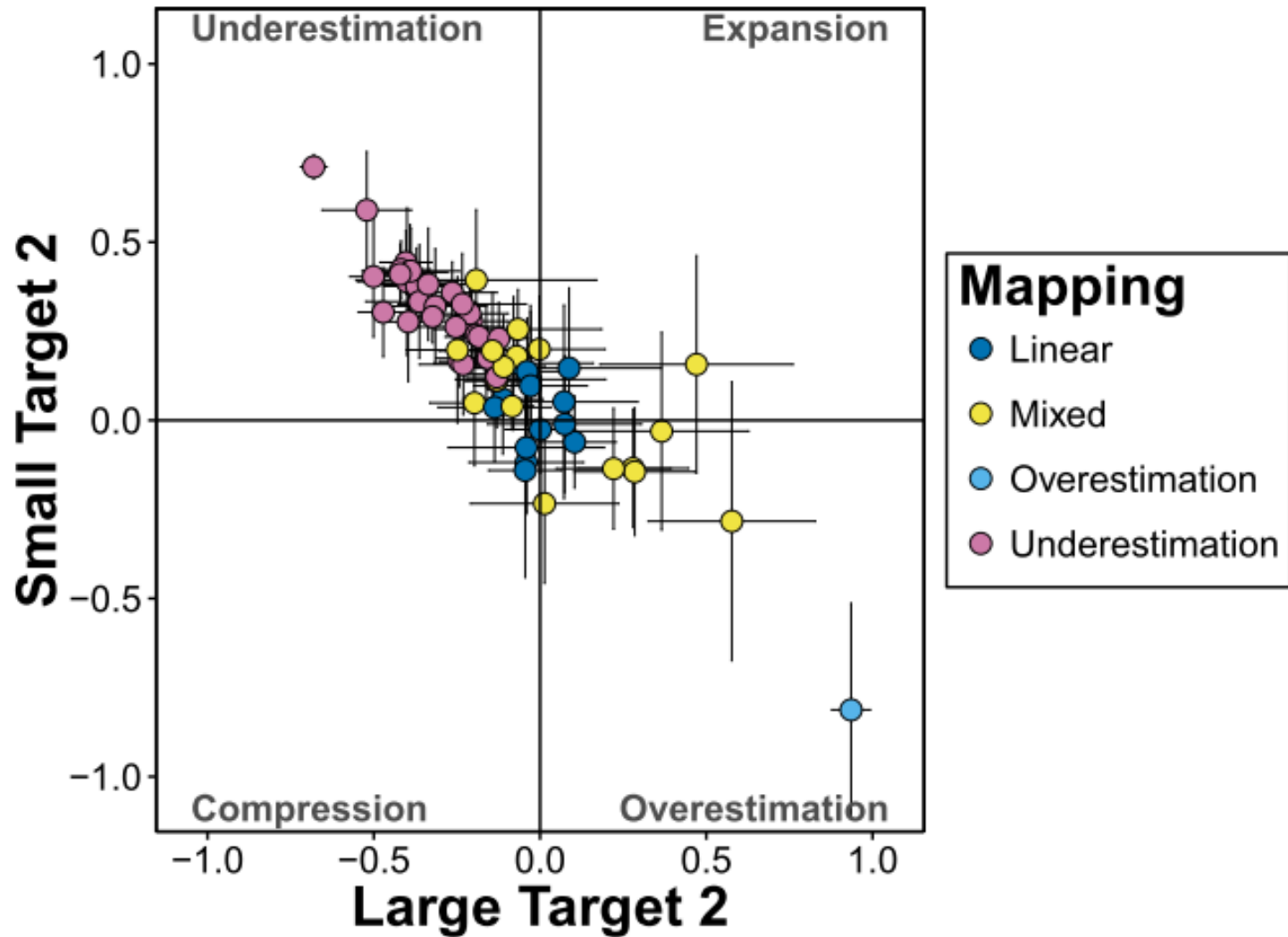


- Overestimation of large **target 2**

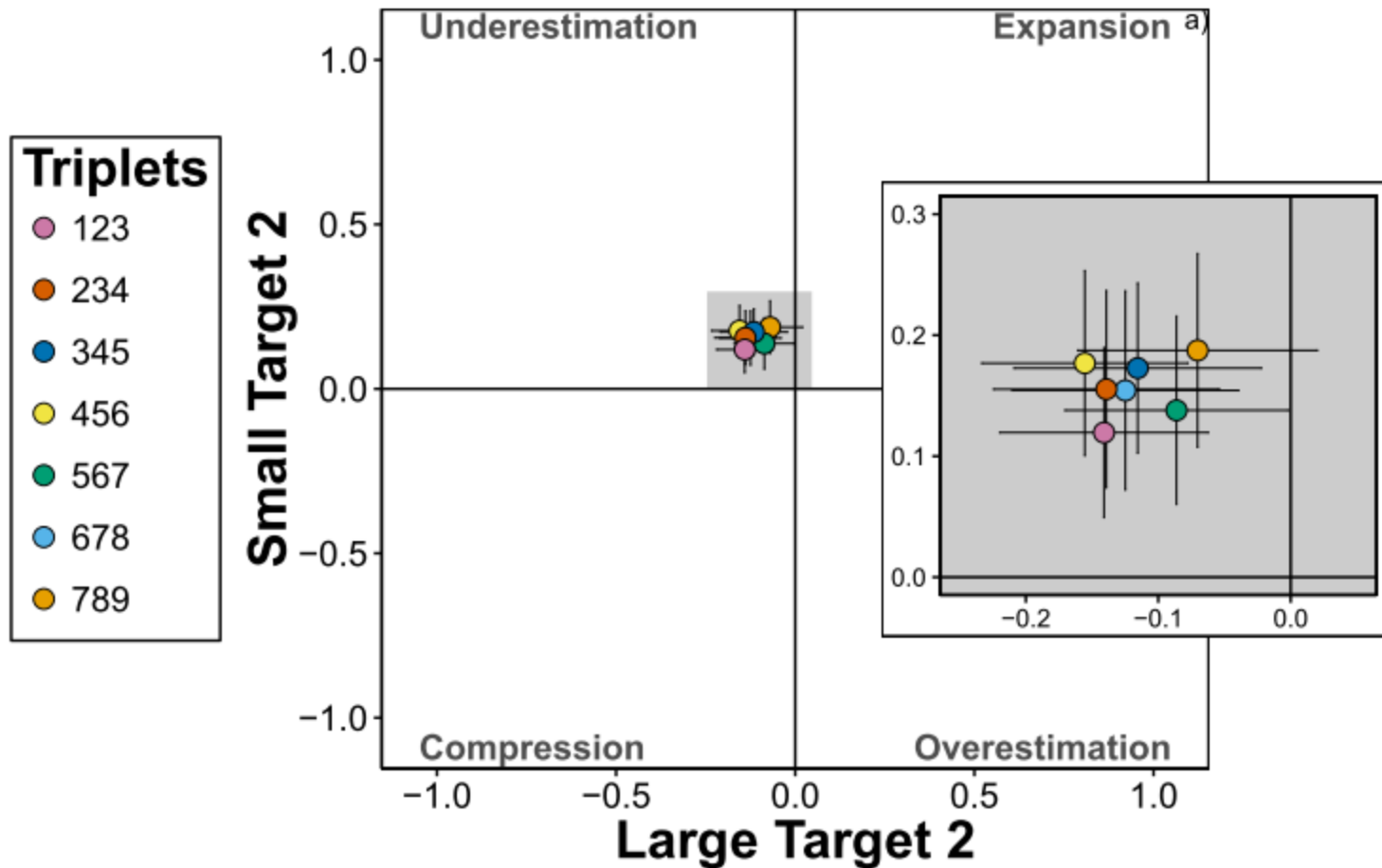


Overestimation
mapping

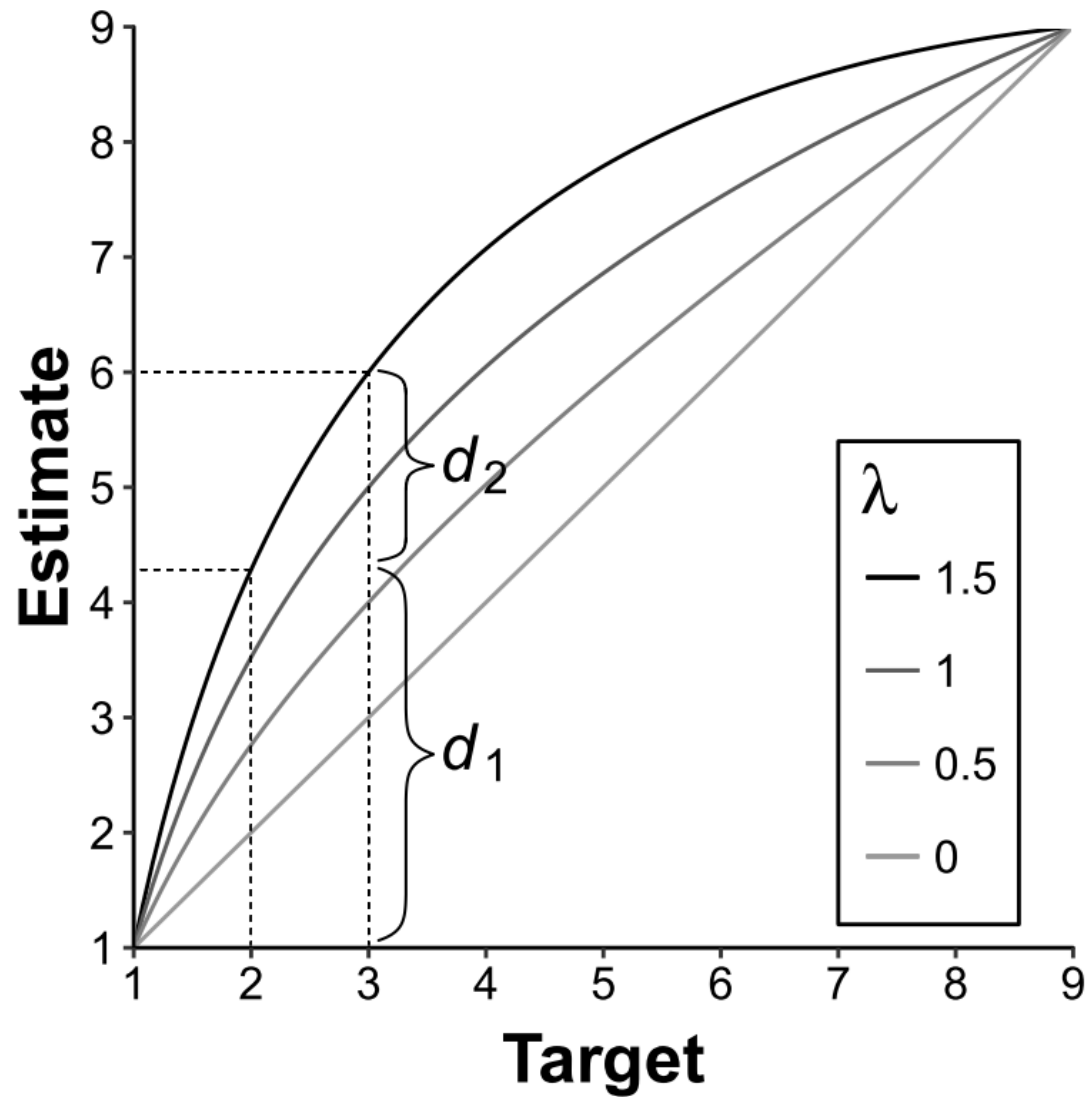
Spatial mapping of digits



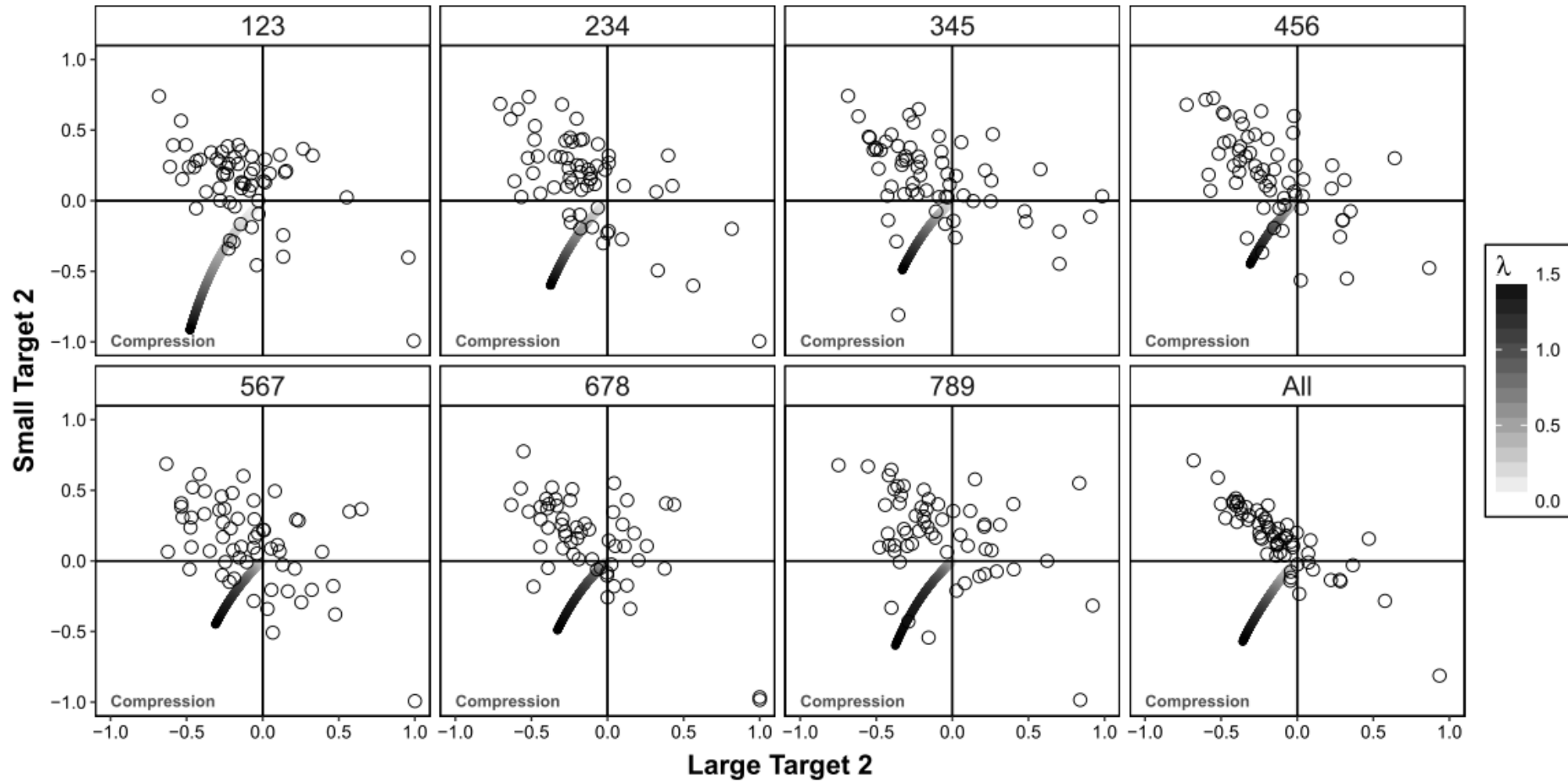
Spatial mapping of digits



Spatial mapping of digits

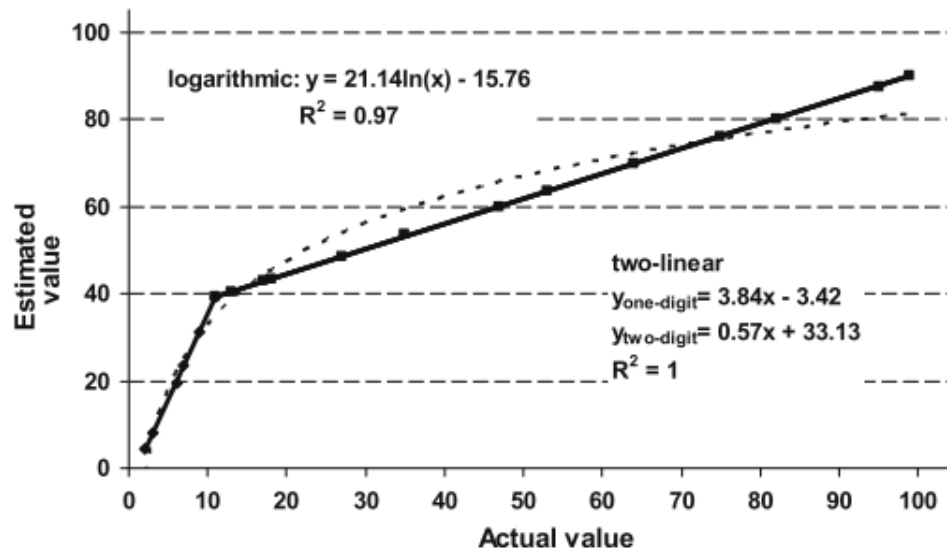


Spatial mapping of digits



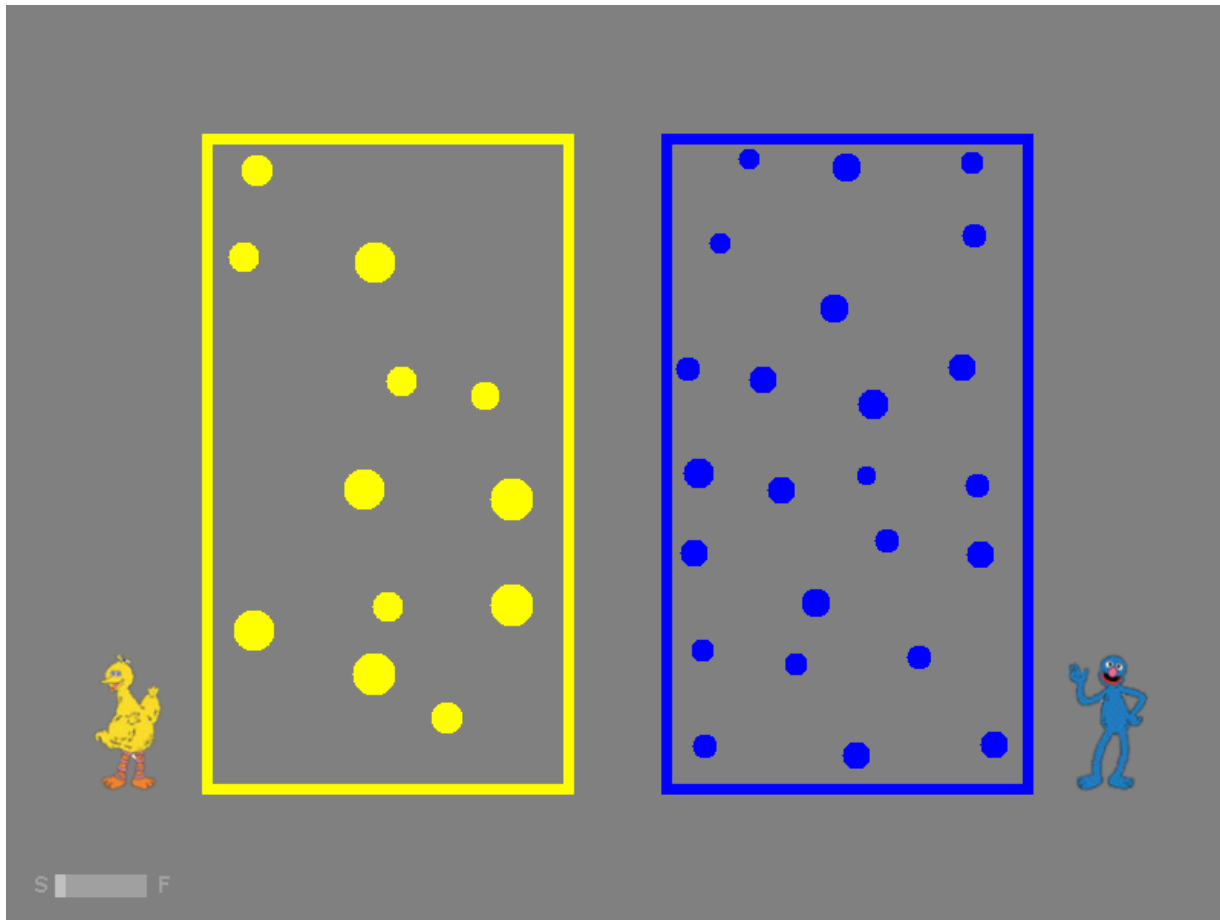
Conclusions

- Ordinality decreased with large digits.
- No evidence of a compressed mapping was found for correctly order digits → **Underestimation pattern.**



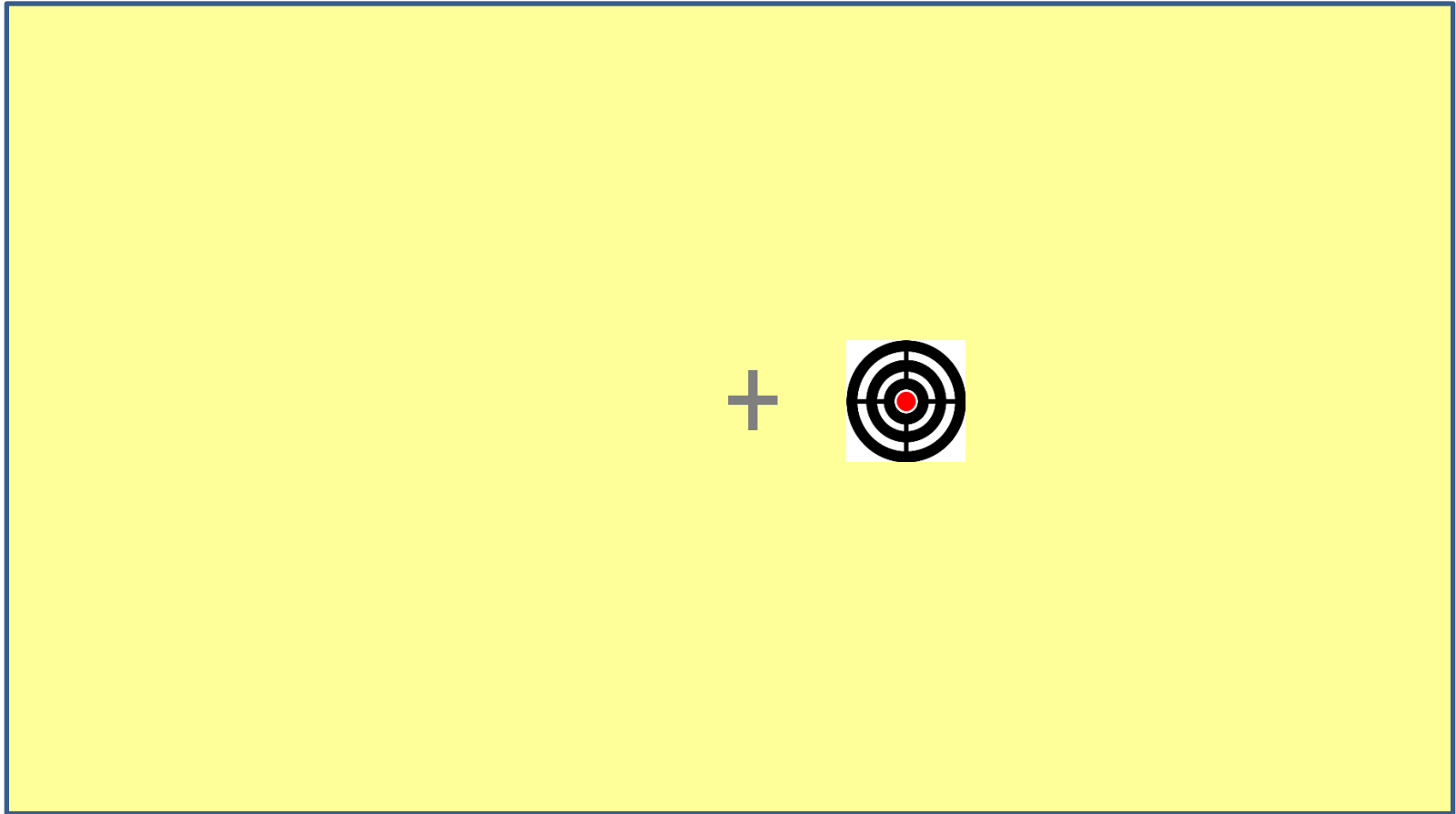
Evidence for the Familiarity model (Moeller et al. 2009)

Simple dot comparison



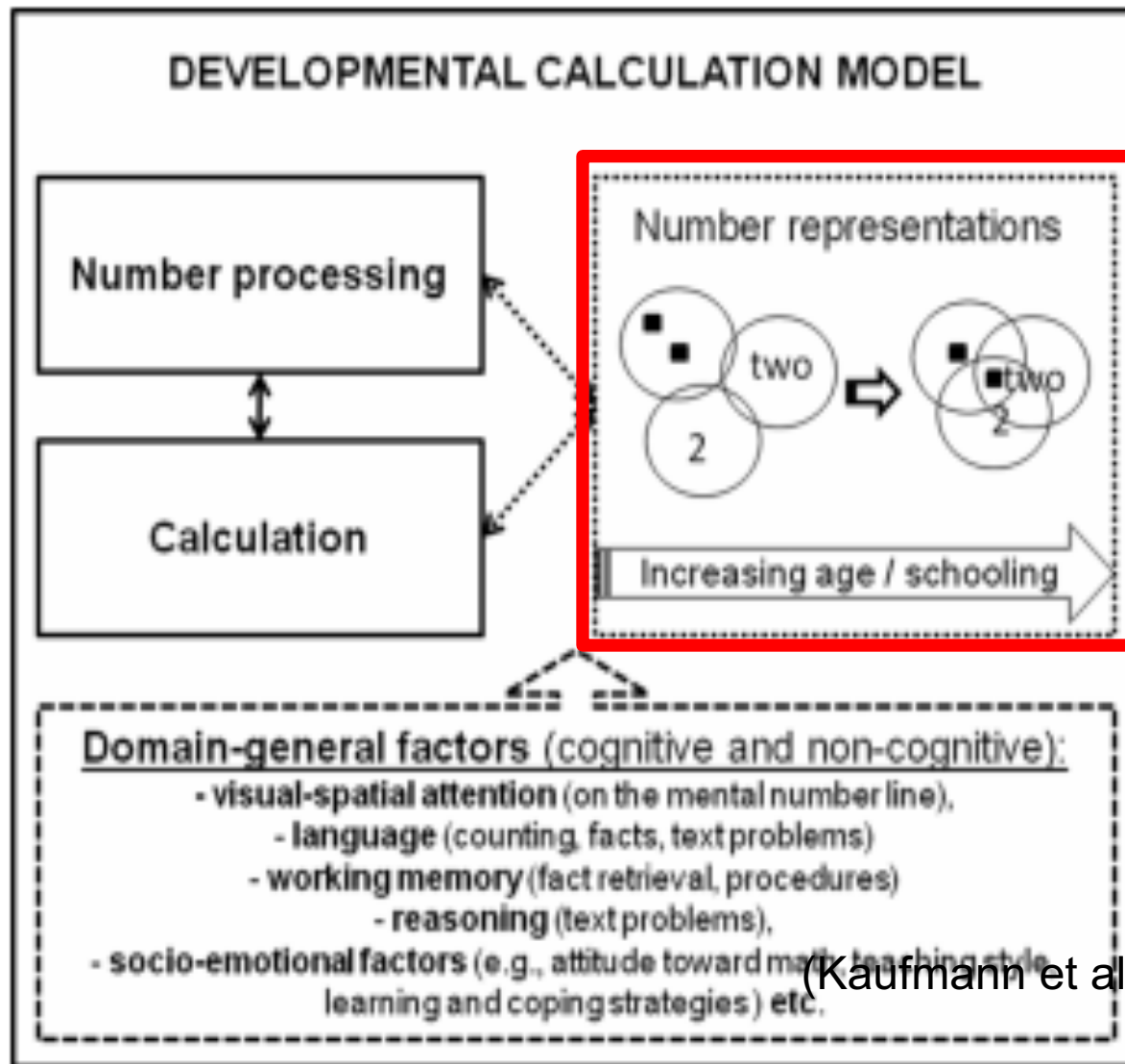
All children understood the meaning of numerically «more»

Aim the target



All children could properly use the computer mouse

Modelli di sviluppo



(Kaufmann et al., 2011)

Counting and spatial mapping of numbers

GaN task \ NL task	Type of mapping 1-10 interval		
	Non-numerical	Logarithmic	Linear
Subset-knowers	23	1	0
CP-knowers	11	3	8

Table 3. Type of mapping in the NL task separated for Subset-knowers and CP-knowers. Cell values represent number of children (N = 46) in Experiment 2.

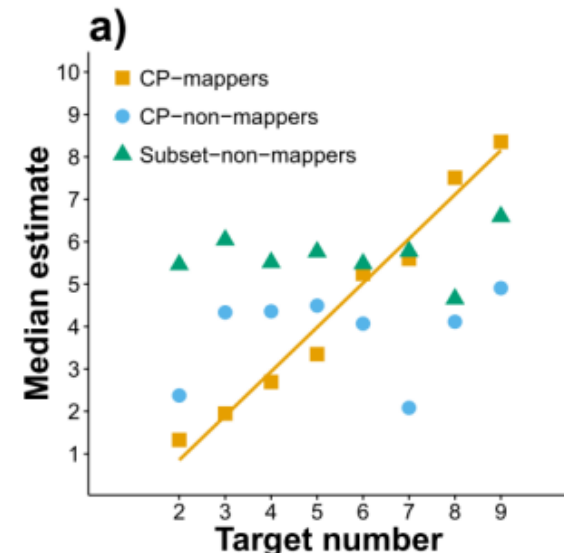














Table 4

Groups	Main analysis						Multiple comparisons							
	SSnMAP	CPnMAP	CPMAP				SSnMAP vs. CPnMAP				CPnMAP vs. CPMAP			
Measures	M(SD)	M(SD)	M(SD)	χ^2	<i>p</i>	η^2	<i>Z</i>	<i>p</i>	BC	<i>r</i>	<i>Z</i>	<i>p</i>	BC	<i>r</i>
Verbal counting (% correct)	51(30)	86(18)	93(13)	18.62	<.001	.42	-3.18	.001	*	.39	-0.93	.35		.14
Naming task 1-10 (% correct)	26(30)	95(10)	96(8)	28.51	<.001	.65	-4.18	<.001	*	.51	-0.45	.654		.07
Naming task 11-20 (% correct)	5(7)	12(24)	38(40)	9.93	.007	.23	-0.64	.524		.08	-2.04	.041		.31
NL interval 1-10 (PAE)	47(5)	43(8)	17(9)	24.02	<.001	.55	-1.36	.173		.17	-4.27	<.001	*	.64
NL interval 1-20 (PAE)	46(6)	41(7)	18(7)	26.39	<.001	.60	-2.04	.042		.25	-4.39	<.001	*	.66
Digit comparison (% correct)	54(15)	61(20)	96(5)	20.85	<.001	.47	-0.98	.325		.12	-3.54	<.001	*	.53
Age (months)	50(8)	63(7)	69(6)	26.31	<.001	.60	-3.71	<.001	*	.45	-1.91	.056		.29

Table 4. Results of statistical analyses comparing Subset-non-mappers (SSnMAP), CP-non-mappers (CPnMAP) and CP-mappers (CPMAP) on numerical tasks and age. Bonferroni correction (BC) adjusted one tail alpha level to .10/14. BC: * $p < .006$. The effect sizes were calculated as: $\eta^2 = \chi^2/N-1$; $r = Z/\sqrt{N}$ (Rosenthal, 1991).

Counting and spatial mapping of numbers

	Cardinality principle	Spatial mapping of numbers	Numerical knowledge	Digit comparison
Subset-non-mappers				
CP-non-mappers				
CP-mappers				

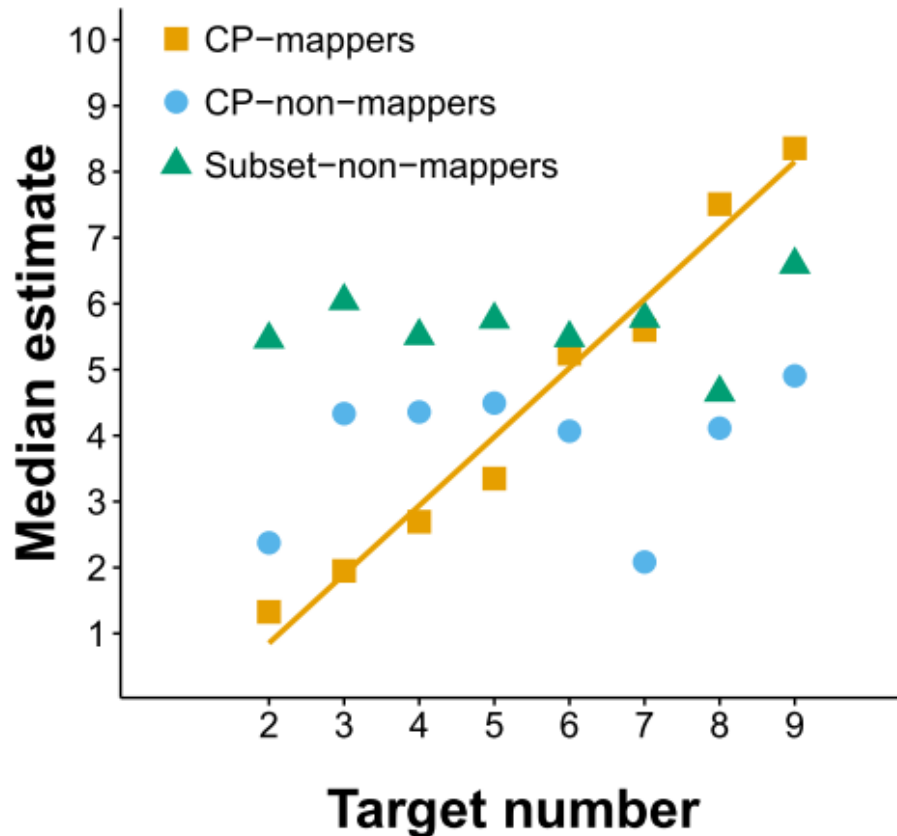
Conclusions:

- Three developmental stages: Subset-non-mappers, CP-non-mappers and CP-mappers.
- CP precedes but does not guarantee a spatial mapping of numbers.
- **Spatial mapping principle:** congruent (i.e., numerically ordered) positioning of numbers onto a visual line as well as the concept that moving rightwards conveys an increase in numerical magnitude.
- The spatial mapping principle might guide children in understanding the magnitude relations between digits.

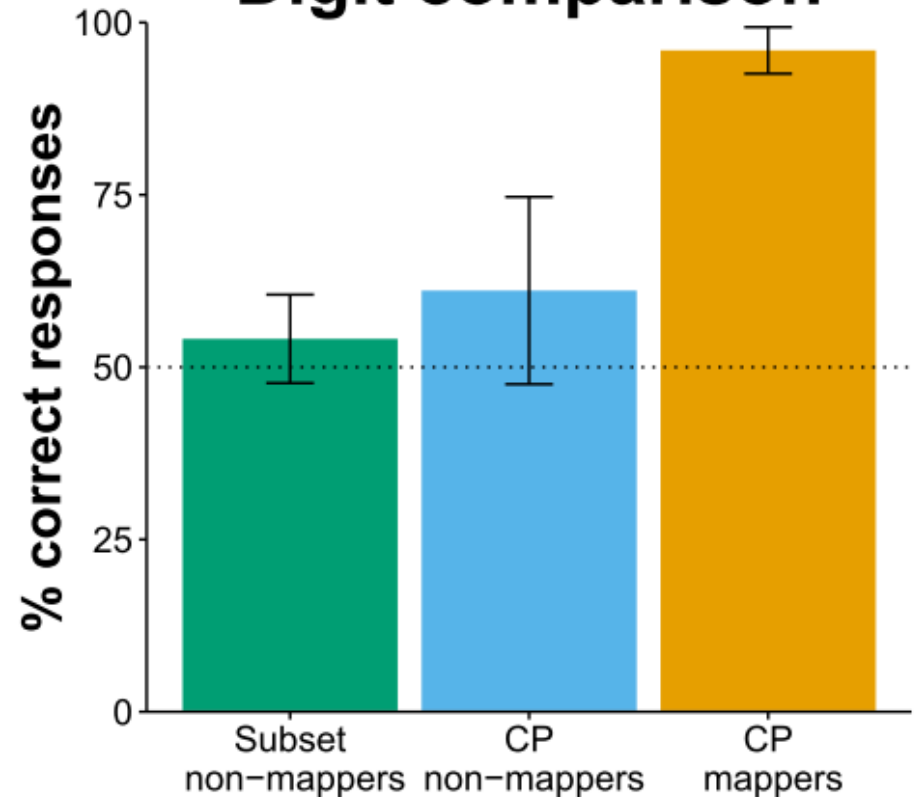
Counting and spatial mapping of numbers

Sella, Berteletti, Lucangeli, & Zorzi (2017).

Number Line 1–10



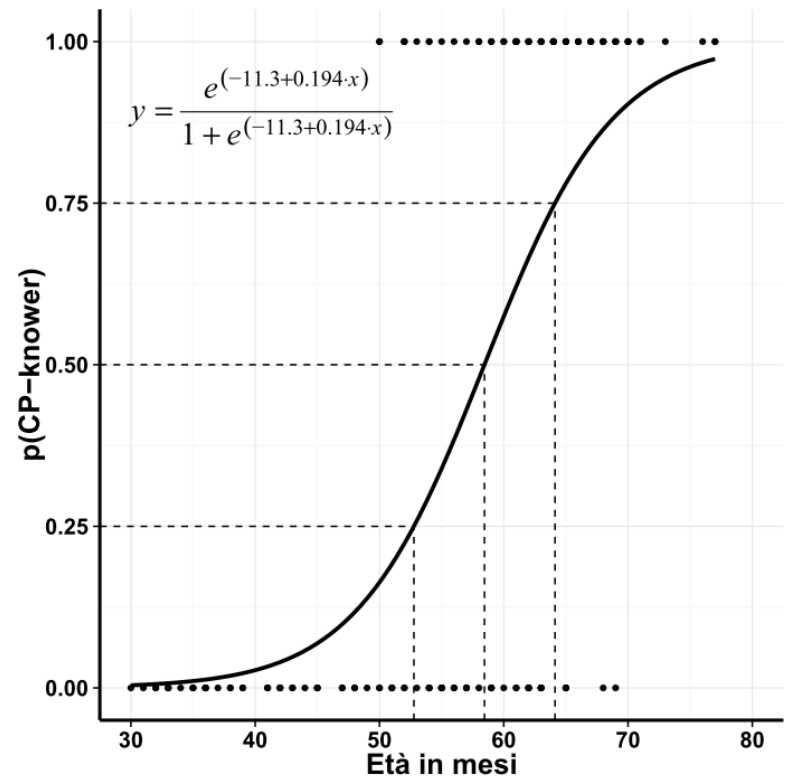
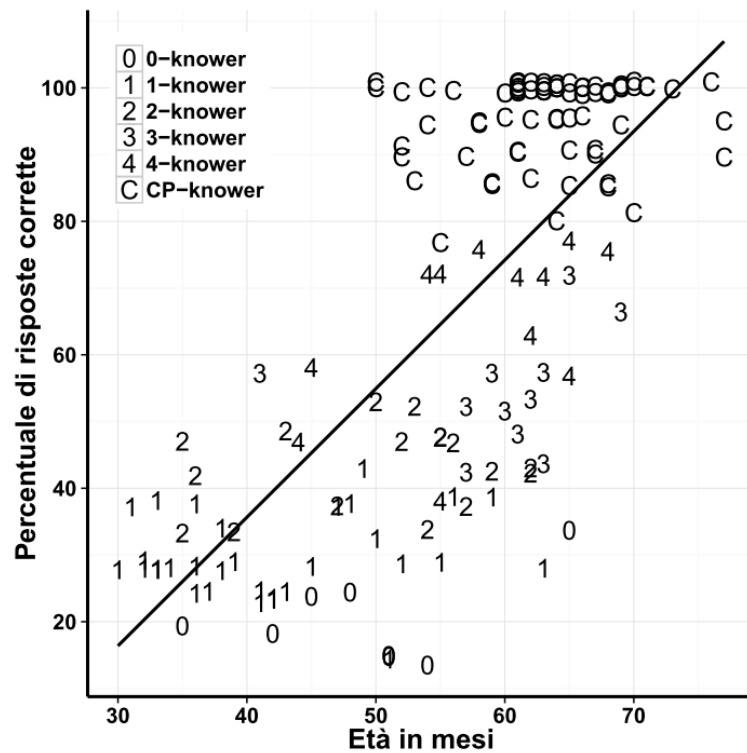
Digit comparison



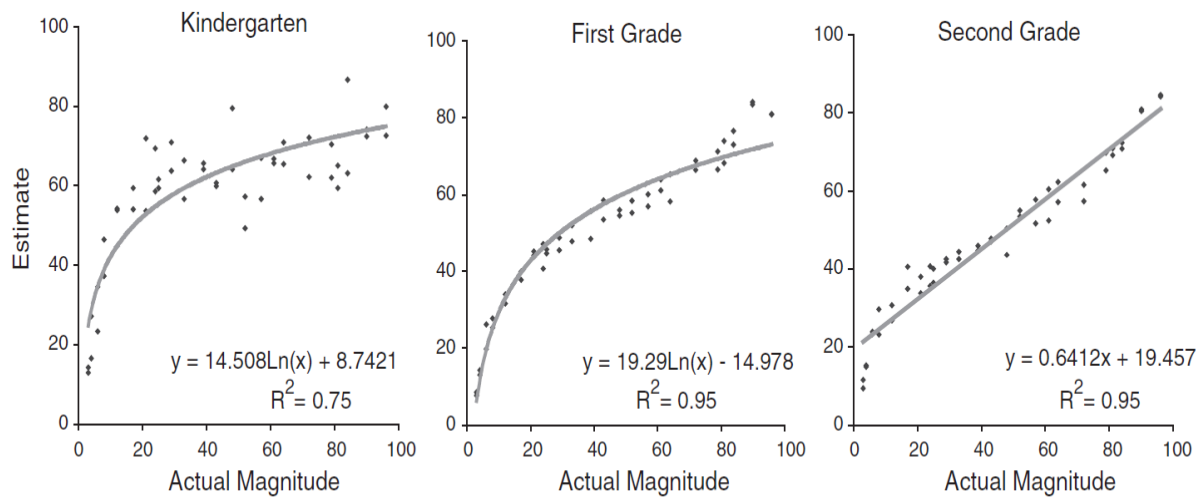
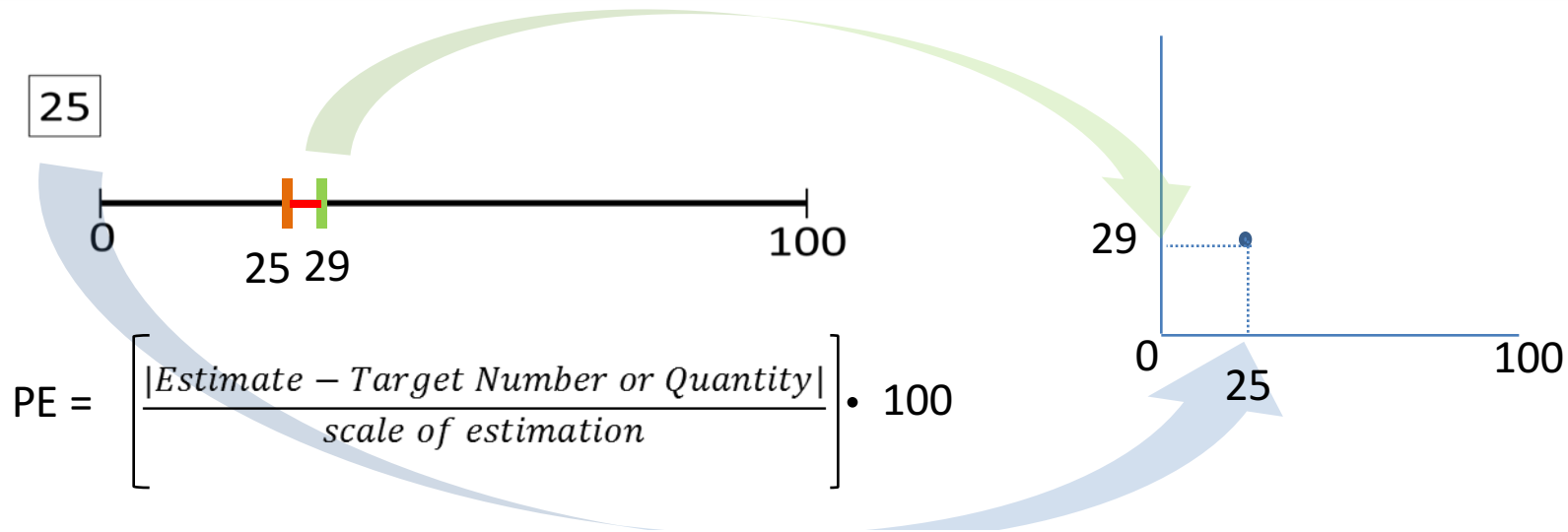
Spatial mapping principle

Knower-level

Sella, Lucangeli, & Zorzi (in revisione) -> Knower level su un campione italiano.



Spatial mapping: Number line task

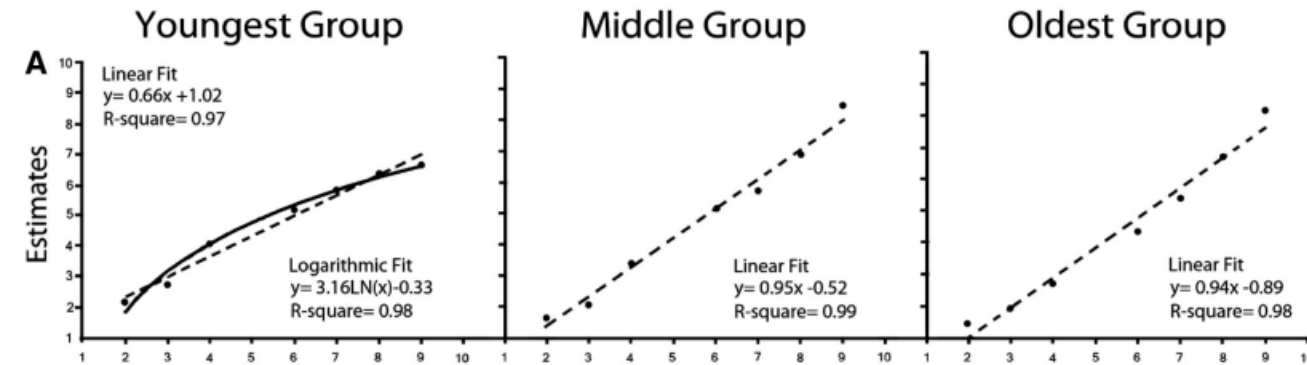


- Logarithmic to linear shift
- Familiarity model

Figure 2. Progression from logarithmic pattern of median estimates among kindergartners (left panel) to linear pattern of estimates among second graders (right panel) in Experiment.

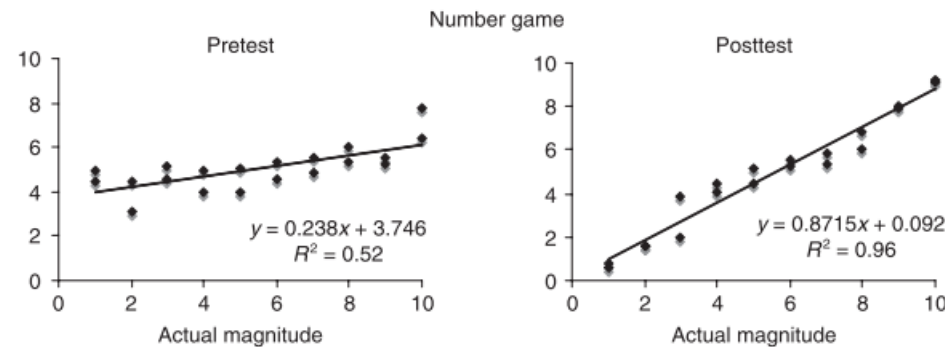
Number line task interval 1-10

Berteletti et al. (2010)



Log-to-lin shift

Siegler & Ramani (2008)



No bias log-like component.

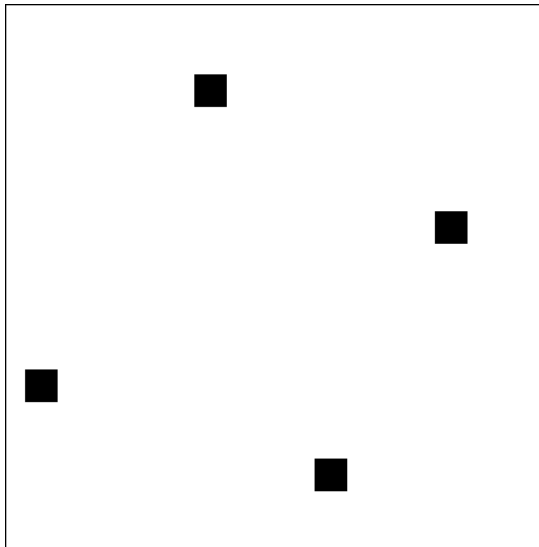
From 5 years of age -> Linear mapping (Sasanguie, De Smedt, Defever, & Reynvoet, 2012a; Sasanguie, Van den Bussche, & Reynvoet, 2012; Simms, Muldoon, & Towse, 2013; Mooldon et al., 2013)

Numerical estimation

Numerosities: 1, 2, 3, 4, 6, 8, 10.

Measure: Absolute difference for small (≤ 4) and large (>4) numerosities.

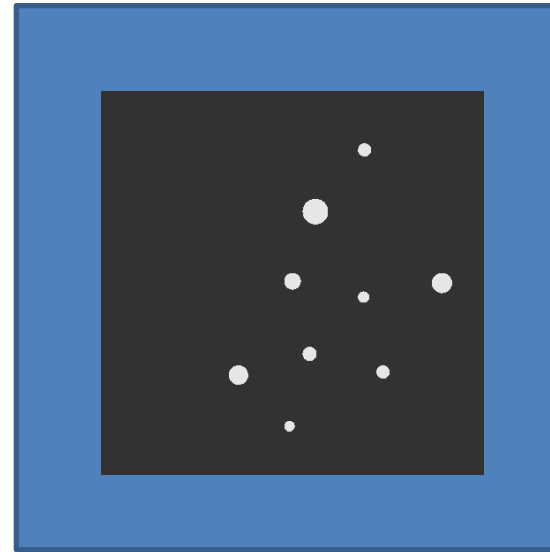
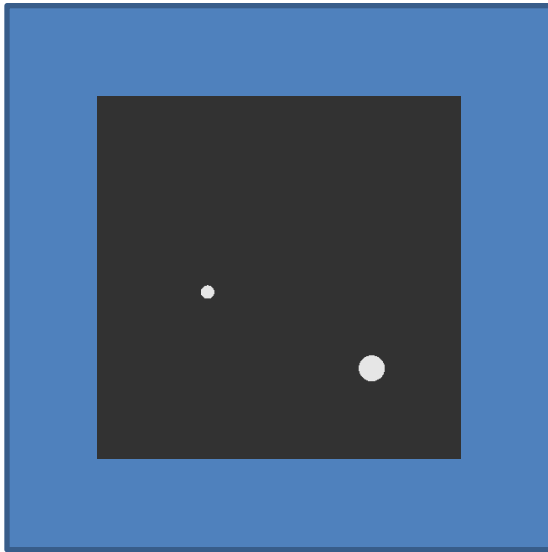
1 second.



Verbal digit comparison


12 randomly presented comparisons (i.e., 1-2, 1-4, 1-6, 1-8, 2-3, 2-9, 3-6, 3-8, 4-9, 6-7, 6-9, 8-9) repeated twice.

“In this box [pointing to the left box] there are 2 balls and in this box [pointing to the left box] there are 9 balls. Which box has more balls?”



Visual digit comparison

72 randomly presented comparisons with all digits from 1 to 9.



2 9

Digit comparison

