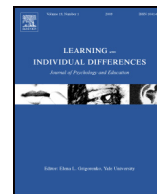




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## Working memory and domain-specific precursors predicting success in learning written subtraction problems

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### ABSTRACT

This study was designed to identify individual differences predicting competence in solving written subtractions with borrowing in second-grade schoolchildren. To examine the role of domain-general and domain-specific precursors, a group of 68 second-graders was tested at three different sessions. Domain-general precursors were analyzed during the first session, including four working memory (WM) tasks, distinguishing between simple-storage and complex-span WM tasks. The domain-specific mathematical abilities tested were knowledge of symbols, arithmetical fact retrieval, understanding of the positional value of digits, and alignment skills. During the second and third sessions, children were taught written subtraction algorithms, first without and then with borrowing procedures, and were then immediately assessed on their acquired competences. Path analysis models were run and the final model showed that performance in written subtractions with borrowing was predicted by both visuospatial WM and specific mathematical skills. The results are discussed for their theoretical and educational implications.

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### 1. Introduction

Acquiring written calculation skills is a fundamental goal of the primary school arithmetical curriculum, and also represents a difficult point of transition essential to learning more complex mathematical skills at higher education levels (Dowker, 2003).

The goal of the present study was to analyze the factors that can predict the successful acquisition of a specific written arithmetic algorithm, i.e. multi-digit subtraction with borrowing, that is typically taught in second grade. Learning this algorithm represents not only one of the main goals of the arithmetic classes for this age group, but also one of the crucial difficulties encountered by primary school children (Selter, 2001; Venneri, Cornoldi, & Garuti, 2003). To date, subtraction has not been thoroughly studied. Most research in the field of arithmetic learning has focused on addition or multiplication. Even less has been done to examine the domain-general and domain-specific competences associated with the ability to solve subtraction problems in primary school (Barrouillet, Mignon, & Thevenot, 2008; Lemaire & Calliès, 2009; Robinson, 2001; Torbeyns, Verschaffel, & Ghesquière, 2004).

#### 1.1. Domain-general competences

A sizable body of research (see e.g. Fuchs et al., 2010a, 2010b) has shown that a number of domain-general competences may predict success in learning at school, particularly in arithmetic. In the present study, we focused on the case of working memory (WM), which has already emerged as an essential aspect of numerical cognition (DeStefano & LeFevre, 2004; LeFevre, DeStefano, Coleman, & Shanahan, 2005; Noël, Désert, Aubrun, & Seron, 2001; Passolunghi & Lanfranchi, 2012; Raghobar, Barnes, & Hecht, 2010). The concept of WM refers to a set of processes or structures that are intimately associated with many arithmetical processes. In the present research, we distinguished between simple-storage and complex WM span tasks, which are considered as distinct constructs (Kintsch, Healy, Hegarty, Pennington, & Salthouse, 1999; Miyake, Friedman, Rettinger, Shah, & Hegarty, 2001). Simple storage tasks only involve storing information (e.g. remembering a series of numbers or letters), while complex WM span tasks entail both storing and processing information (Kail & Hall, 2001; Miyake & Shah, 1999). We also distinguished between verbal and visuospatial WM processes because an increasing amount of research has shown that different WM components are associated with arithmetic at different ages (Krajewski & Schneider, 2009; McKenzie, Bull, & Gray, 2003). Visuospatial WM appears to be crucially implicated in the mathematical performance of younger children who are still learning basic arithmetical skills (see also Bull, Espy, & Wiebe, 2008; Holmes, Adams, & Hamilton, 2008; Maybery & Do, 2003; McKenzie et al., 2003; Rasmussen & Bisanz, 2005).

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In the present study, we concentrated on the period when children learn multi-digit written subtraction with borrowing. This stage can be clearly identified in the life of a primary school class and it requires explicit teaching (VanLehn, 1990). It is a stage that may be associated with potentially different predictors from those relating to other aspects of arithmetic. In fact, a number of experimental studies have shown that the role of WM components in calculation depends on several factors, such as the type of algorithm required, the presentation format, or the involvement of carrying/borrowing procedures (Ashcraft & Kirk, 2001; DeStefano & LeFevre, 2004; Trbovich & LeFevre, 2003). The verbal WM component seems to be involved in addition and multiplication problems (Seitz & Schumann-Hengsteler, 2002), while the visuospatial one seems to relate to subtractions (Lee & Kang, 2002), though this may be mediated by cultural effects (Imbo & LeFevre, 2010). Other studies found that operations with carrying or borrowing procedures increased the demands on WM, prompting a selective involvement of WM components (Caviola, Mammarella, Cornoldi, & Lucangeli, 2012; Imbo, Vandierendonck, & Vergauwe, 2007; Mammarella, Lucangeli, & Cornoldi, 2010).

### 1.2. Domain-specific competences

During their formal education, children gradually learn procedures for using multi-digit algorithms. The mathematics curricula in many countries conventionally distinguish between three main arithmetical approaches: written standard algorithms (procedural knowledge, also called the routine approach), written informal algorithms (making notes or using equations), and mental arithmetic, or the strategic approach (that involves applying strategies drawn from a given individual's strategy repertoire) (Heinze, Marschick, & Lipowsky, 2009; Selter, 2001). Multi-digit arithmetic is still taught mainly with paper-and-pencil written standard algorithms, however (Selter, Prediger, Nührenbörger, & Hussmann, 2012). A mathematics curriculum for teaching arithmetic that is based on the written standard approach requires that children in grade 1 (typically 6 years old) consolidate their counting skills and start learning the principles of adding and subtracting, while the procedures for solving written additions and subtractions are taught (in that order) in grade 2 (Cornoldi & Lucangeli, 2004).

It is worth noting here that, when children perform calculations, they allocate their cognitive abilities differently, not only according to the type of algorithm required, but also to the problem's complexity (Imbo et al., 2007), and they may use specific strategies (Robinson, 2001; Seyler, Kirk, & Ashcraft, 2003). Some general assumptions can nonetheless be made on the main processes involved in learning subtraction. For instance, having learnt to deal with addition problems presumably helps children to learn subtractions, partly because the latter are explained in terms of the addition procedure in reverse (Selter et al., 2012; Torbeyns, De Smedt, Stassens, Ghesquière, & Verschaffel, 2009). Solving a multi-digit subtraction problem according to the written standard approach involves splitting the operands into digits that are then manipulated using explicitly prescribed procedural rules. In particular, the standard algorithm taught in the classroom requires that pupils know how to align the operands correctly and then process single columns, working from the unit (on the right) to the tens (on the left), and applying special rules to borrow ten when the bottom digit is larger than the top digit. This means that multi-digit subtraction demands a conceptual understanding of the meaning of symbols, place value and the base-ten number system, as well as the ability to identify quantities, encode and transcribe quantities in an internal representation code, and keep track of partial results while completing the next step. Other arithmetical abilities that are probably important for subtraction include an efficient fact retrieval and a good expertise in mental calculation (Butterworth, 2005; Dowker, 2005).

### 1.3. The present study

The present study was designed to shed further light on the precursors and mediators involved when children learn written subtractions by examining the role of individual differences in both the domain-specific and the domain-general precursors (i.e. numerical and procedural knowledge in the former case, simple and complex visuospatial and verbal WM components in the latter) that take effect when written subtractions are taught using the standard method. We followed up five classes of 2nd-grade pupils throughout a school year, testing them on three different occasions (at the start of the school year, in November, and in February). In the first session, we assessed variables that we assumed might predict the acquisition of subtraction with borrowing. Regarding the domain-specific abilities, we hypothesized that the acquisition of the subtraction algorithm is linked to competences in the calculation domain, such as arithmetical fact retrieval, and proficiency in solving written addition problems. We also hypothesized that more specific abilities in the number domain, e.g. the expertise needed to process judgments of magnitude or understand the value of each digit in a complex number, would affect a child's acquisition of the subtraction procedure. As for abilities in the general domain, we predicted a different influence of WM components. In particular, we assumed that performing subtractions would mainly engage visuospatial WM resources, while the influence of verbal WM would emerge as a mediator of the child's knowledge of the principles of addition.

In the second and third sessions we assessed arithmetical learning in terms of written arithmetic that we assumed might mediate the relationship between the predictors considered and the acquisition of subtraction with borrowing (which was also measured in the third step). For the mediators, we assumed that the acquisition of written additions, both simple and with carrying, and simple subtractions would be supported by the same predictors as those supporting complex subtractions (Dowker, 2005; Fuchs et al., 2006), and further involved in the acquisition of subtraction with borrowing. The acquisition of subtraction was assessed immediately after conducting two lessons on written subtraction procedures (one without and one with borrowing) to exclude any confounding influence of any other variables intervening between the acquisition of the subtraction algorithms and any later assessment. The teacher's role was controlled by asking the class teachers not to teach subtraction, and arranging for 2 h of structured lessons to be conducted by one and the same teacher in all the classes.

## 2. Method

### 2.1. Participants

The study concerned 96 primary school pupils (60 M, 36 F) tested in 2nd grade. The children were attending 5 different classes at schools in north-eastern Italy. They were tested over the course of three sessions, starting from the beginning of the academic year. Parental consent was obtained beforehand for all the children. Children with certified special educational needs, intellectual disabilities or neurological/genetic disorders, or from families with a very low socio-economic status were excluded. At the start of the study, the mean age of the children was 93.04 months ( $SD = 4.6$ ). For inclusion in the final model for analysis, data on a given child had to be available for all three sessions, so any child without a full dataset was excluded from the statistical analyses. The final complete sample consisted of 68 pupils (44 M, 24 F).

### 2.2. Tasks and procedure

The first session (S1) was completed in two phases. In one, the children were collectively administered a series of mathematical tasks adapted from a standardized arithmetic battery (Lucangeli, Tressoldi, & Fiore, 1998). In the other, they were individually tested on verbal and visuospatial simple-storage and complex WM span tasks.

Two different lessons on the written subtraction algorithm were held by the same teacher, who gave exactly the same lesson to all the classes. The first lesson was about simple written subtraction problems and was given after the children had learned the simple addition algorithm (Session 2). The second lesson was on written subtractions with borrowing and was given after simple subtraction and complex addition had been taught (Session 3). Each lesson lasted approximately 45 min and, after a short break, the children were asked to solve 10 simple written subtractions and 10 simple written additions in Session 2, and 10 written subtractions with borrowing and 10 written additions with carrying in Session 3. As no standardized measure of written calculation with a sufficient number of problems is available in Italy, the problems were devised by the Authors with the support of an expert in teaching arithmetic.

### 2.2.1. Arithmetical academic achievement

The children were presented with paper-and-pencil tasks adapted from a standardized arithmetic battery (Lucangeli et al., 1998). Cronbach's alpha for the reliability of this battery is .78. The tests included: a) retrieving combinations and numerical facts to test the children's number fact knowledge (*fact retrieval*); b) arranging series of 5 numbers from the smallest to the largest, and vice versa (*number ordering*); c) writing in Arabic format a series of numbers spoken aloud by the experimenter (*number dictation*); d) naming and explaining the meaning of the main arithmetical symbols +, −, >, and < (*symbol naming*); e) aligning two multi-digit numbers in columns (*alignment*); f) judging magnitude, i.e. children chose the larger of two numbers (*magnitude judgment*); g) recognizing a digit's place value, i.e. children had to establish the meaning of each digit in relation to the position it occupied, e.g. 256 = 2 hundreds, 5 tens, 6 units (*place value*); and h) transcription (transcoding) from place value to Arabic numerals, which involved children recomposing a number starting from place value indications, i.e. 2 hundreds, 5 tens, 6 units = 256 (*transcoding*). The children's performance (mean correct answers) in each task was considered as the dependent variable.

### 2.2.2. Written arithmetic

Twenty addition problems, first without and then with carrying (*simple and complex additions*), and 20 subtraction problems, first without and then with borrowing (*simple and complex subtractions*) were devised. The problems were 2- and 3-digit operations presented horizontally on a squared sheet. The children were asked to solve the problems using the written algorithm procedure. The children received no further instructions and there were no time constraints. Children who completed the task quickly were allowed to use colors to embellish the task sheet. The number of correctly-answered problems for each type of algorithm was considered as the dependent variable. The Cronbach reliability coefficients are higher than .83 for each set of problems.

### 2.2.3. Simple-storage WM tasks

**2.2.3.1. The Syllable Span Test (*syllable span*).** In this test (adapted from Mammarella et al., 2006), lists of syllables varying from 2 to 9 syllables long (three trials for each length) are presented orally at a rate of one syllable per second. Participants are asked to listen and then repeat the list of syllables in their order of presentation. The Cronbach reliability coefficient is .82.

**2.2.3.2. The Corsi Blocks test (*Corsi test*).** The Corsi Blocks test (Corsi test) (adapted from Corsi, 1972; Mammarella, Toso, Pazzaglia, & Cornoldi, 2008) consists of a set of nine blocks placed at random on a board. The task involves remembering a series of locations in the same order as they were presented, without any further processing of the material (which is why this was considered a simple storage task). The examiner taps some of the blocks at a rate of one per second in increasingly long

random sequences, and participants are asked to tap the same sequences of blocks in their order of presentation (i.e. in the forward sense). There are three trials for each length of the sequence (involving from 2 to 8 blocks). The reliability coefficient is .79.

For the two tasks, the scores corresponded to the length of the longest correctly recalled lists and sequences.

### 2.2.4. Complex-span WM tasks

**2.2.4.1. Categorization Working Memory Span Task (*categorization*) (CWMS; De Beni, Palladino, Pazzaglia, & Cornoldi, 1998).** The materials consist of sets of 2 to 6 lists, each containing five words of medium-to-high frequency, some of them animal names. For instance, a set of 2 word lists could contain: (i) house, mother, dog, word, night; and (ii) car, table, sun, frog, book. Participants are shown the words on a computer screen, presented at a rate of 1 s for each word, and are asked to press the spacebar whenever they see an animal name. The interval between two lists of words is 2 s. At the end of a set of lists, participants are asked to recall the last word on each list in the order of their presentation (i.e. night and book in the above example). The Cronbach reliability coefficient is =.71. The participant's score is the mean percentage of correctly recalled words.

**2.2.4.2. Dot matrix task (*dot matrix*) (adapted from Miyake et al., 2001).** The dot matrix task involves participants having to imagine joining two lines together while remembering an increasing number of dot locations in a 5 × 5 grid. For each display, participants are asked to check the outcome of joining the two lines, then they are administered the memory test. The number of dot locations to recall in a given sequence ranges from 2 to 6. The Cronbach alpha is =.79. The score is the mean percentage of correctly recalled positions.

## 3. Results

Preliminary analyses showed that gender and class did not significantly affect performance in any of the arithmetical tasks, so these variables were not taken into account. Descriptive statistics for each test are given in Table 1, while the correlations between the measures are shown in Table 2.

### 3.1. Model estimation

Path analysis models were computed using the LISREL 8.7 statistical package (Jöreskog, & Sörbom, 1996).

Since path analysis relies on the assumption of multivariate normality of the observed data, the normality issue was considered in a preliminary step. For this purpose, Mardia's measure of relative multivariate kurtosis (MK) was obtained with the PRELIS program (Jöreskog & Sörbom, 1993). The MK was 1.06 for the present sample, a value implying no significant departure from normality ( $-1.96 < Z < 1.96$ ) (Mardia, 1970).

We used the fit indices recommended by Jöreskog and Sörbom (1993), such as the root-mean-square error of approximation (RMSEA), the non-normed fit index (NNFI), and the comparative fit index (CFI). In agreement with Schreiber, Stage, King, Nora, and Barlow (2006; see also Schermelleh-Engel, Moosbrugger, & Müller, 2003), we considered substantively interpretive the models with a non-significant chi-square, an RMSEA below .05, an NNFI above .97, and a CFI above .97 as a good fit. We also assumed that a model that has a good fit and a small Akaike information criterion (AIC) provides the best description of the relationships between variables (Bentler, 1990; Schermelleh-Engel et al., 2003).

In our analysis, we tested a model based on the assumption that: the acquisition of written subtraction with borrowing is a complex multifaceted learning process in which both WM and basic number competences may be precursors; and the acquisition of more simple addition

**Table 1**  
Descriptive statistics.

	Tasks	M	SD	Skewness	Kurtosis	
Domain-specific	Arithmetical academic achievement	Number dictation	7.118	0.923	-1.061	1.065
		Fact retrieval	8.956	2.645	-0.818	-0.019
		Magnitude judgment	5.456	1.165	-2.665	6.835
		Transcoding	4.206	1.570	-1.330	1.842
		Number ordering	8.294	2.649	-1.897	2.694
		Symbol naming	4.735	1.589	-0.124	-0.846
	Written arithmetic	Place value	7.309	1.557	-2.270	4.008
		Alignment	5.824	2.491	-0.759	-0.783
		Simple additions	8.176	2.533	-1.828	3.127
		Additions with carrying	5.000	3.095	-0.028	-1.022
		Simple subtractions	7.132	3.124	-0.938	-0.2
		Subtractions with borrowing	4.765	3.940	-0.08	-1.668
Domain-general	Simple storage	Syllable span	4.397	0.715	0.5	0.102
		Corsi test	4.074	0.739	0.566	1.854
	Complex span	Categorization	45.806	12.072	0.01	-0.643
		Dot matrix task	58.224	15.581	-0.697	0.159

and subtraction operations may mediate the relationship. We therefore examined whether the best model emerging from the analysis would include WM measures (assessed at session 1) as well as measures concerning arithmetical operations (assessed at session 2), or only the specific arithmetical precursors assessed at session 1. In the analysis, the independent variables were the four WM tasks and the measures derived from the mathematical achievement tasks drawn from the standardized battery. The mediator variables concerned the competences related to the previously-acquired written addition and subtraction algorithms, i.e. the number of correctly-solved simple additions and subtractions, and complex additions. The number of correctly-solved subtractions with borrowing was the dependent variable.

We started the analysis by assessing the fully saturated model, which included the relationships between all the variables (see Fig. 1). The fully saturated model was tested on the matrix of covariance between the study variables using the maximum likelihood method of estimation (Jöreskog & Sörbom, 1996). We gradually removed the non-significant relationships starting from the lowest *t* value and changing one path at a time. In proceeding with the analysis, we first examined the direct relationships between the independent and dependent variables, the direct relationships between the independent variables and the mediator variables, and the relationships between the mediators and the dependent variable. During this process, some of the independent variables were deleted when all the paths departing from them were set to zero, according to their weight and significance, and our hypotheses.

We report only the results of the three principal models in detail, as they best illustrate the overall analytical process.

The first model that we tested (Path Model 1) included all the variables considered in the study, based on the assumption (in light of the literature) that they could predict the acquisition of the written subtraction algorithm. The model was saturated, with no non-significant correlations between the independent variables. The fit was perfect:  $\chi^2(0, N = 68) = .001, p = 1.00, NNFI = 1.00, CFI = 1.00, RMSEA = .0001$  (see Fig. 1).

Path Model 2 summarizes all the steps that led to the omission of direct paths between the independent variables and the dependent variable. Starting from the weakest bond, the paths involving the following independent variables were dropped in the following order: number dictation, magnitude judgment, Corsi test, number ordering, transcoding, categorization, alignment, symbol naming, syllable span, and the place value task. The only direct paths remaining were: fact retrieval, the dot matrix task, and the number of correctly-solved simple subtractions, simple additions and additions with carrying. As several other direct relationships between independent variables and mediators were still not significant, we kept deleting one path at a time, always choosing the weakest relationship. The following paths were dropped in the following order:

- for additions with carrying: symbol naming, number ordering, Corsi test, fact retrieval, and dot matrix task;
- for simple additions: magnitude judgment, categorization,

**Table 2**  
Correlations between all variables.

	A. Written calculation				B. Mathematical tasks								C. WM tasks		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A.1. Subtractions with borrowing	1														
A.2. Simple subtractions	.55**	1													
A.3. Additions with carrying	.47**	.37**	1												
A.4. Simple additions	.52**	.76**	.44**	1											
B.5. Number dictation	.37**	.32**	.28*	.30*	1										
B.6. Fact retrieval	.55**	.56**	.39**	.52**	.55**	1									
B.7. Magnitude judgment	.34**	.29*	.31**	.31**	.26*	.36**	1								
B.8. Transcoding	.21	.29*	.34**	.33**	.14	.30**	.15	1							
B.9. Number ordering	.48**	.50**	.28*	.40**	.43**	.55**	.55**	.21	1						
B.10. Symbol naming	.11	.27*	.22	.06	.26*	.31*	.08	.17	.01	1					
B.11. Place value	.24*	.40**	.41**	.52**	.10	.29*	.27*	.22	.24	.27*	1				
B.12. Alignment	.28*	.47**	.34**	.37**	.01	.30*	.24*	.18	.32**	.11	.43*	1			
C.13. Corsi test	.16	.25*	.11	.17	-.17	-.04	.29*	.08	.31*	-.08	.11	.26*	1		
C.14. Syllable span	.26*	.15	.26*	.20	.18	.18	.05	.03	.06	.16	.04	.07	.00	1	
C.15. Dot matrix task	.42**	.32**	.20	.38**	.14	.23	.40**	.15	.49**	.02	.40**	.38*	.42**	.03	1
C.16. Categorization	.14	.14	.15	.19	.04	.30*	.18	.13	.25*	.09	.14	.22	.10	.28*	.12

\* *p* < .05.  
\*\* *p* < .01.



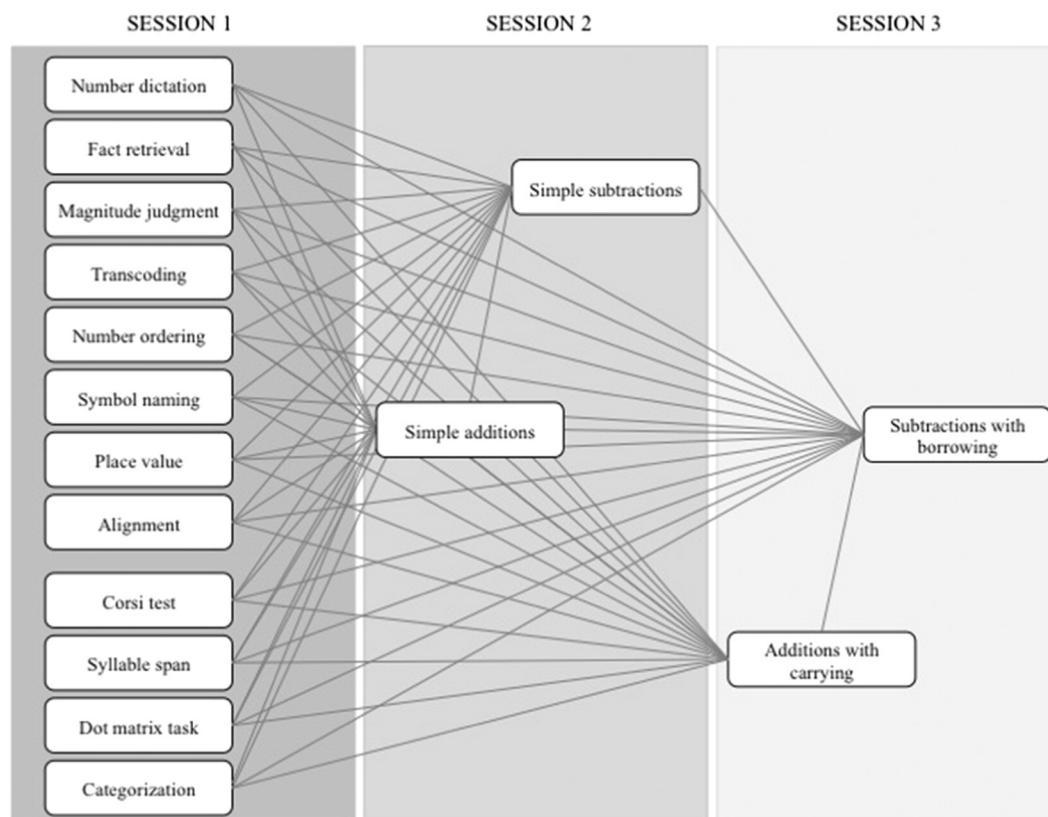


Fig. 1. Conceptual diagram of Path Model 1.

alignment, number ordering, dot matrix task, and number dictation;

- for simple subtractions: number dictation, transcoding, syllable span, fact retrieval and magnitude judgment.

The fit indices of the model were  $NNFI = 1.16$ ,  $CFI = 1.00$ , and  $RMSEA = .0001$ , and the chi-square was not significant  $\chi^2 (72, N = 68) = 11.80$ ,  $p = 1.00$  (see the weights of the predictors in Table 3).

In Path Model 3, which was actually developed from Path Model 2, all the non-significant indirect effects on complex subtractions were deleted, along with the independent variables that remained isolated (number dictation, magnitude judgment, transcoding, Corsi test and categorization). In this final model (see Fig. 2), as we had expected, both domain-general and domain-specific abilities predicted success in performing subtractions with borrowing. The place value ( $\beta = .14$ ) and syllable span ( $\beta = .04$ ) tasks, and the number of correctly-solved simple additions ( $\beta = .20$ ) predicted performance in subtractions with borrowing, mediated by the number of correctly-solved additions with carrying. Number ordering ( $\beta = .05$ ), symbol naming ( $\beta = .05$ ), and alignment ( $\beta = .04$ ) also predicted the score obtained for subtractions with borrowing, mediated by the simple subtractions. Finally, fact retrieval ( $\beta = .28$ ) and the dot matrix ( $\beta = .24$ ) tasks, along with simple subtractions ( $\beta = .23$ ) and additions with carrying ( $\beta = .23$ ), directly predicted performance in solving written subtraction problems with borrowing. The direct relationship between simple subtractions and subtractions with borrowing neared statistical significance. The fit indices of the model were very good ( $NNFI = 1.00$ ,  $CFI = 1.00$ ,  $RMSEA = .0001$ ) and the chi-square was not significant  $\chi^2 (32, N = 68) = 22.22$ ,  $p = .90$ . This model not only explained a relevant proportion (47%) of the variance in the scores for subtractions with borrowing ( $R^2 = .47$ ), but also provided a satisfactory description of the factors affecting the acquisition of this arithmetical procedure (see Fig. 2).

#### 4. Discussion

This study focused on the contributions of precursors – both domain-specific (numerical and procedural knowledge) and domain-general (WM components) – in children learning written subtractions, when the algorithm is taught according to the standard procedural methods. Children in 2nd grade were tested three times during the school year to obtain a sort of ongoing picture of their acquisition of written subtractions.

Our final path analysis model was able to explain a relevant proportion of the variance relating to the acquisition of complex subtraction skills. In particular, our findings revealed a direct link between the scores in the complex visuospatial WM span task and written subtraction performance, consistent with the hypothesis that visuospatial WM is involved in solving subtractions – for the correct alignment of numbers in columns, for instance, and for retaining and completing the borrowing procedure (which includes several sequential steps in a strict order and direction, and has specific spatial constraints). Our results support previous research on the relationship between visuospatial WM and arithmetical performance (Caviola et al., 2012; Heathcote, 1994; Raghubar et al., 2010; Trbovich & LeFevre, 2003), confirming a specific role for visuospatial WM in subtraction problems. In fact, it has been demonstrated in adults that performing a secondary visuospatial task may interfere with the solving of simple subtractions, but not with multiplications (Lee & Kang, 2002; Seitz & Schumann-Hengsteler, 2002).

The relationship between visuospatial WM and written subtraction also seems to be affected by the type of arithmetical task, as well as by the individual's age. Correlational studies have shown that visuospatial competences might be engaged in the early learning of new mathematical skills/concepts, while the phonological loop may come into play after the skills have been learned (Bull et al., 2008; Holmes et al., 2008; and for a different pattern of results, see also De Smedt et al., 2009; Meyer, Salimpoor, Wu, Geary, & Menon, 2010). Our findings

**Table 3**  
Direct and indirect effects predicting performance in subtractions with borrowing, and total standardized regression weight ( $R^2$ ) in Path Models 1 to 3.

Dependent variable	Independent variable	Direct effect		Indirect effect		Total $R^2$
		$\beta$	Z	$\beta$	Z	
<i>Path Model 1</i>						
Subtractions with borrowing	Simple subtractions	.26	1.45			.53
	Additions with carrying	.26	2.23*			
	Simple additions	.24	1.72	.20	1.56	
	Number dictation	.05	.37	.05	.92	
	Fact retrieval	.40	2.69*	.13	1.69	
	Magnitude judgment	.03	.23	.01	.28	
	Transcoding	.03	.26	.08	1.53	
	Number ordering	.10	.63	.04	.51	
	Symbol naming	-.06	-.50	.02	.23	
	Place value	-.01	-.04	.12	1.58	
	Alignment	.04	.30	.10	1.61	
	Corsi test	.04	.33	.07	1.33	
	Syllable span	.20	1.82	.08	1.49	
	Dot matrix task	.24	1.79	-.04	-.62	
Categorization	-.11	-.96	-.05	-1.00		
<i>Path Model 2</i>						
Subtractions with borrowing	Simple subtractions	.21	1.36			.48
	Additions with carrying	.22	2.01*			
	Simple additions	.03	.22	.17	1.52	
	Number dictation			.03	1.00	
	Fact retrieval	.28	2.31*	.08	1.49	
	Magnitude judgment			.03	.90	
	Transcoding			.07	1.76	
	Number ordering			.05	1.22	
	Symbol naming			.01	.21	
	Place value			.10	1.56	
	Alignment			.07	1.54	
	Corsi test			.04	1.30	
	Syllable span			.07	1.74	
	Dot matrix task	.23	2.22*	-.03	-.95	
Categorization			-.02	-.95		
<i>Path Model 3</i>						
Subtractions with borrowing	Simple subtractions	.23	1.97			.47
	Additions with carrying	.23	2.27*			
	Simple additions			.20	2.59*	
	Fact retrieval	.28	2.51*			
	Number ordering			.05	1.57	
	Symbol naming			.05	1.64	
	Place value			.14	2.66*	
	Alignment			.04	1.42	
	Syllable span			.04	1.39	
	Dot matrix task	.24	2.39*			

\*  $p < .05$ .\*\*  $p < .01$ .

suggest that the phonological loop is involved in the acquisition of arithmetic algorithms, but to a lesser extent than active visuospatial WM. It is worth noting, however, that the other two WM tasks considered in our study did not appear to predict performance in written arithmetic. In the case of the Corsi task, this may be because visuospatial WM is involved in arithmetic when information must be not only maintained but also processed, while the Corsi task only assesses the maintenance function (Mammarella, Pazzaglia, & Cornoldi, 2008). In the case of categorization span, our results confirm that verbal WM is less crucial to performance in written arithmetic and its involvement may be limited to the phonological component. Future research might take a similar approach to ours, but examine the performance of older children to investigate whether the role of WM components in solving written subtractions differs in individuals with a more conscious and mature application of the borrowing procedure.

As for the domain-specific skills, the present study showed that a crucial part was played by knowledge of arithmetical facts, which related directly to subtractions with borrowing (see also Campbell, 2008; Seyler et al., 2003; Thevenot & Castel, 2012). This means that a good knowledge and retrieval of arithmetical facts enables intermediate

results to be calculated more quickly during arithmetic problem solving. The child may thus avoid the need to use less-functional processes, based on counting down or the fingers, which make the calculation slower and divert cognitive resources away from the monitoring of the different written calculation steps.

When the indirect effects on subtractions with borrowing were considered, both simple subtractions and additions with carrying emerged as significant mediators. This means that the difficulties children might encounter in learning the complex subtraction algorithm involved in borrowing can be avoided by a well-established prior knowledge of how to solve additions with carrying and simple subtraction problems (VanLehn, 1990). Written subtractions with borrowing require a number of competences that are involved in additions too, but are more challenging in many respects. Unlike the case of additions, for example, the commutative property cannot be applied in subtractions and the order of the subtracting terms coming into the sequential procedures does make a difference. That is why simple additions, simple subtractions, and additions with carrying were entered in the model as mediators, and the relationships between simple subtractions and additions with carrying were significant, masking the effects of the other

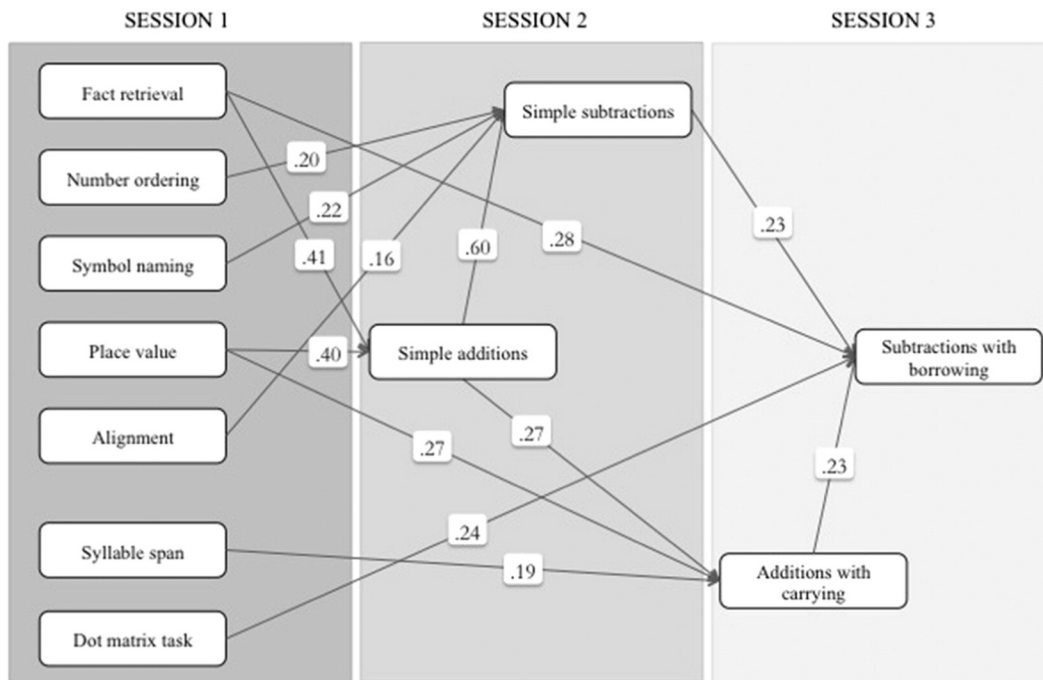


Fig. 2. Standardized solution of the final Path Model 3.  $\chi^2 (32, N = 68) = 22.22, p = .90, NNFI = 1.00, CFI = 1.00, RMSEA = .001$ .

domain-specific variables; on the other hand, simple additions did not mediate the effects on subtractions with borrowing, but they were related with both simple subtractions and additions with carrying.

The role of domain-specific math precursors was crucial when it came to simple subtractions and additions with borrowing. In particular, performance in simple subtractions was explained not only by the number of correctly-solved simple additions, but also by semantic and visuospatial arithmetical skills (e.g. number ordering, alignment and symbol naming). This picture is consistent with several other studies reporting that early numbering and mathematical skills can be seen as a prerequisite for complex arithmetical reasoning (Jordan, Kaplan, Oláh, & Locuniak, 2006; Krajewski & Schneider, 2009; Passolunghi & Lanfranchi, 2012). At the same time, performance in additions with carrying was found supported by expertise in simple additions and by syntactic numerical knowledge, as used in place value tasks. These results support previous observations. In a longitudinal study on first- through third-graders, Hiebert and Wearne (1996) found a close relationship between children's understanding of multi-digit numbers and their level of computational skill. Their results showed that children who developed an understanding of place value and base ten concepts early on (in 1st grade) performed at the highest level on computational tasks at the end of their 3rd grade. It is not only an early understanding of the number system that supports mathematical learning throughout primary school, however; domain-general skills also help to explain performance in additions with carrying, as also suggested by several previous studies (DeStefano & LeFevre, 2004; Imbo et al., 2007; Kalamian & LeFevre, 2007; Noël et al., 2001) showing a significant relationship between general phonological abilities and specific math achievement.

To sum up, our results showed that both domain-general and domain-specific competences predict the acquisition of written multi-digit operations, and that an earlier acquisition of arithmetical algorithms can mediate the relationship between predictors and the acquisition of one of the most delicate arithmetical algorithms, i.e. subtraction with borrowing.

Before any conclusion can be drawn from our results, it is important to consider the limitations of the present study. The main issue concerns the small sample size, due partially to the number of participants dropping out in the various stages of the research. Further research

should replicate our final model in a larger sample. A second limitation concerns the choice of assessment procedure. This study focused on the specific period when the written subtraction procedure is taught and we did not examine its automatization and consolidation. Another limitation concerns the choice of precursors considered in the initial model and the tasks associated with them. For instance, we did not consider approximate number system acuity, which has been shown to correlate with performance in a broad mathematical achievement test in 3rd-grade children, after controlling for various domain-general ability measures (Halberda, Mazocco, & Feigenson, 2008; see also Fuchs, Geary, Compton, Fuchs, Hamlett, et al., 2010a, 2010b). Another limitation of our study relates to the analysis of other domain-general factors. We only considered the role of verbal and visuospatial WM, but other crucial domain-general mechanisms involved in math performance might be relevant too. For example, future research should examine the role of general fluid intelligence, processing speed, executive functions, and particularly updating and inhibitory control (Ackerman, 1988; Bull & Lee, 2014; Lubinski, 2000; Walberg, 1984).

## 5. Conclusion

The results of the present study have not only theoretical but also educational implications. They show that some children's difficulties in learning arithmetical operations can be foreseen, and possibly prevented, by assessing a number of crucial precursors. In particular, children's difficulties in solving written calculations may relate to both WM weaknesses and specific arithmetical mind bugs. Particular attention should therefore be paid to supporting children in difficulty – especially when arithmetical calculations become increasingly complex and demand greater WM resources – by reinforcing their fact retrieval and mental calculation skills in order to reduce the load on their WM (see Gathercole, Alloway, Willis, & Adams, 2006), and by consolidating the acquisition of simpler arithmetical algorithms before moving on to more complex ones.

In conclusion, exploring the precursors of written subtraction learning in 2nd-grade children helps us to draw a picture of how subtraction knowledge develops. This study suggests that both domain-specific and domain-general competences are involved. In particular, both visuospatial WM and fact retrieval abilities are directly related to the learning of



written subtraction procedures with borrowing, while different general and specific competences support the learning of simple subtraction and addition. Moreover, our results indicate that other specific concepts involved in additions with carrying and in simple subtractions play an important part when it comes to mastering subtractions with borrowing.

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