Arithmetic Education and Learning Disabilities in Italy

Cesare Cornoldi and Daniela Lucangeli

Abstract

In the first part of this article, we briefly present the context of mathematics education in Italy and its specific programs, teaching and assessment methods, use of psychological concepts, and policies in favor of pupils with difficulties. We illustrate the importance and the role of mathematics teachers' associations. In the second part, we focus more directly on the issue of arithmetic learning and learning disabilities in Italy. Despite the absence of a specific strong tradition in the field, new perspectives regarding mathematics education are evident in Italy. The preparation of new assessment tools has proceeded together with an effort toward the identification of subtypes of arithmetic learning disabilities. A basic distinction has been made between difficulties in number processing/calculation and problem-solving tasks. Within the area of calculation, a finer distinction between numerical knowledge, speed, and accuracy seems critical, but a more articulated differentiation of subcomponents relying on modern cognitive–neuropsychological models can be used also.

In the first part of this article, we summarize the debate on the role of mathematics in educational projects for Italian children. We analyze both the Italian context of mathematics education and the influence of psychological models in teaching and learning mathematics.

Italian Context of Math Education

Italy has a long tradition of studies on mathematics education. However, projects in this field started in the universities only at the beginning of the 1970s. The debate enhanced the cultural dimension of science (which for a long time had been considered culturally inferior to *les belles lettres*) and offered a new image of mathematics by reducing its technical aspects and increasing its more general implications. In the last 25 years, mathematicians interested in educational methods started to interact, to have frequent meetings, and to develop general ideas on the teaching of mathematics, creating a body of Italian research into mathematics education (RME).

As illustrated by Arzarello (1992, 1996), four main components of RME can be identified, and the current research represents an original integration and elaboration of these components. The first three components may be considered an Italian representation of general trends in the international scientific research community, and the fourth represents the current integrated perspective. The components are as follows:

1. RME based on the conceptual organization of the subject (mathematics);
2. RME for concrete innovation in the classroom;
3. RME as observation and modeling of laboratory processes; and
4. research for innovation.

Conceptual Organization of Mathematics

The aim of this kind of RME is to improve the teaching of mathematics in generic situations, working on the logical organization of concepts in mathematics. Attention is focused on the contents. Didactic interventions in the classroom are planned by taking into consideration students' difficulties related to mathematics only. The objects of study are the teaching products rather than the teaching and learning processes.

One application of such perspective is the syllabus that was published by the *Unione Matematica Italiana* (UMI) in 1980. It provided a hierarchic description of the main concepts and abilities that students should master at the end of secondary school, showed the main difficulties and mistakes in current education, and proposed challenging problems.

Concrete Innovation in the Classroom

The aim of this kind of RME is to understand how to improve the teaching of mathematics in specific contexts with respect to concrete and peculiar problems emerging in the everyday life of the classroom. Intervention in the classroom is rooted in practice, and great attention is paid to teaching and learning processes, rather than just outcomes.

Thanks to the contributions of prominent scholars (e.g., Castelnuovo, 1965), the most widespread vision of mathematics is based on reality. The basis of didactic strategies consists in starting from the pupils' concrete activity, possibly with ad hoc didactic tools, and going on through problems, the solution of which could help teachers to build up and give significance to mathematics at a more abstract level.

The teacher's role has had to be transformed. Teachers can no longer simply carry out the annual objective, step by step; instead, they must become designers of long-term didactic projects based on precise cultural choices that include a deeper analysis of specific issues contained in the syllabuses.

Observation and Modeling of Laboratory Processes

This RME is focused on understanding the relationship between the teaching and learning processes. It requires the use of methodologies that are borrowed from other disciplines, such as psychology, sociology, and pedagogy. Typically, experiments are prepared, either in a laboratory or in the classroom, which works as a laboratory, in order to test previously formulated hypotheses. The influence of this perspective has been very important, because it was the starting point for a reformulation of Italian mathematics education research and determined the characteristics of the current RME. The interaction among the three components described produced a very original development and evolution of Italian RME, which gave it an interesting international dimension.

Current Trends in Italian RME

The Eighth National Seminar for Mathematics Education, held in Pisa in 1991 (see Malara, Menghini, & Reggiani, 1996), presented the new developments in Italian RME. Researchers realized that there are relationships among mathematical contents, teaching strategies, and learning processes. Interventions in the classroom were planned with attention to cultural and epistemological perspectives, pedagogic and conceptual reasons, cognitive difficulties, and specific interactions in the classroom. Projects for global or local interventions were designed to produce innovations in the curricula (e.g., with respect to contents or teaching methods), models for classroom processes (e.g., the role of the teachers), and so on. These projects were usually framed by a theoretical framework and were the result of the collaboration of teachers and researchers, in both the planning and the observation phases. As described by Arzarello and Bartolini-Bussi (1998), theory and practice were generated and developed at the same time. This methodology of research, which has an empirical basis, represents a crucial epistemological perspective. Avoiding the distinction between observer and observed (in educational research, the observer is represented by the researcher and the observed by the classroom together with the teacher) represents a shift from the traditional positivistic methods, borrowed from natural sciences, which focus on the observed component.

Effects of RME on School Activities

As a result of the aforementioned elaboration, Italian schools are combining, in a variety of different forms, tradition and innovation in teaching mathematics and in approaching mathematical difficulties. This variety has led to the loss of some well-established features that made the Italian system different from that in other countries.

However, some specific features still remain; for example, as illustrated by a comparative study involving the United States, Japan, and Italy (Santagata & Stigler, 2000), building on the Third International Mathematics and Science Study (TIMSS) video study, a consistent proportion of the mathematical teaching time in Italy is still used for oral examination of single pupils in front of the class. This examination is intended as a form of assessment of students' learning (which has the same importance as the written examination) and as an opportunity for the other students to learn from errors and correct responses produced by the examined classmate. Santagata and Stigler suggested that Italian teaching is characterized by distinctive features that may constitute an Italian cultural script for teaching.

The issue as to whether there is such an Italian teaching script is of more than academic concern. The improvement of teaching is rightfully perceived as a pressing need in Italy, as it is in many other countries. Italy has initiated a variety of efforts to improve mathematics learning. For example, the Italian government recently funded the participation of Italian schools in international studies of students' achievement, such as the Intensive Educational Assessment (IEA), Third International Mathematics and Science Study Repeat, and the Program for International Student Assessment.

Influence of Psychological Models in Math

Simultaneously with the development of the debate on RME, the psychological research has offered models and criteria to interpret the role of teaching in mathematical learning. According to Sempio (1997), behaviorism was the first psychological approach applied to mathematical instruction. Three main principles were derived:

- attention to the task analysis of the content,
- taxonomy of objectives, and
- learning described as a list of performances rather than of processes.

The most criticized aspect of this perspective by the RME was the fragmentation of the program without attention to the complexity of mathematical discipline.

Since the beginning of the 1970s, the influence of the work of Piaget has been very strong in educational debate in math. Some main principles were derived from the idea that numerical knowledge depends on the development of intelligence; before the age of concrete operational thinking, cognitive access to the number is impossible. In particular, the cognitive abilities of classification and seriation are at the basis of arithmetical competencies. For example, if a child is not able to order a sequence of sizes, he or she cannot have a complete representation of the cardinality of a number. The influence of this perspective has inspired the set theory (*insiemistica*) that has been strongly used by Italian teachers, which bases mathematics teaching on the comprehension of its underlying logical basis.

In the following years, cognitive psychology researchers (see Lucangeli & Passolunghi, 1995) gave increasing attention to the complexity of learning processes and further changed the perspective from performances to processes and from contents to the dynamic learning and instruction context. Finally, in the last few years, Italian researchers have been paying attention to the specific abilities involved in the main mathematical areas of calculation and arithmetical problem solving.

In the area of calculation, some studies have analyzed the specific cognitive competencies that instruction has to exercise and developed specific instructional procedures, focusing particularly on the strategic abilities for learning number facts and developing mental calculation skills and on the automatization of algorithms necessary for written procedures.

In the area of problem solving, particular attention has been devoted to the mathematical comprehension of the text and its transformation into a mathematical schema of solution. The role of metacognitive competencies—in both learning and in teaching processes—was also analyzed. Furthermore, RME stressed the reciprocal influence of cognitive and emotional aspects involved in learning in a social context, as at school (see Boscolo, 1997). As a result of these studies, metacognitive programs were developed that considered a variety of general aspects, such as the children's ideas about mathematics, math anxiety, and specific control processes like planning, monitoring, and evaluation (see Lucangeli, Cornoldi, & Tellarini, 1998). Although these programs had a general use for typically developing children, they were mainly developed and proposed in the context of mathematics learning disabilities. In the next section, we illustrate how psychological and instructional research on mathematical learning led to specific development in the field of learning disabilities (LD).

Arithmetic Learning and LD In Italy

The aim of the second part of this article is to summarize the Italian debate on the identification of arithmetic learning disabilities (ALD). We mention Italian studies on ALD, give information about the normative data we are collecting to analyze the arithmetic performances of children at elementary school, and present our perspective in regards to identifying subtypes of ALD.

Italian Studies on Mathematical Learning and LD

Among the large number of Italian studies considering mathematical learning and learning disabilities (for a review of these studies up to 1995, see Lucangeli & Passolunghi, 1995), we will focus in this article on the recent ones that have offered an articulation of the learning process. In our view, they are particularly important to the extent that they have been able to suggest coherent consequences for assessment and treatment. In this field, the most relevant contributions concern arithmetic and proceed from a distinction between basic arithmetic (BA) and arithmetic problem solving (APS).

BA has its roots in numerical knowledge and tends to develop automatized specific competencies that can be affected by specific difficulties in children (e.g., Temple, 1991) and specific impairments in adults (e.g., McCloskey, 1992). APS necessarily requires BA, but it also involves more complex cognitive processes. In particular, some Italian studies examined the role in specific difficulties in APS of metacognition (Lucangeli & Cornoldi, 1997) and of the inhibition of irrelevant information (Passolunghi, Cornoldi, & De Liberto, 1999).

A fractionated analysis of the distinct components of APS (comprehension, representation, planning, and evaluation) appeared useful for examining APS in children in primary school (Lucangeli, Tressoldi, & Cendron, 1998, 1999).The dissociation between BA and APS was confirmed by the fact that metacognitive training focusing on controlled processes substantially improved the APS skills of children with ALD but did not affect their BA skills (Lucangeli, Cornoldi, & Tellarini, 1998).

Very recent research was devoted to finding general subtypes of BA disabilities. In particular, this was done in the context of a recent large-scale Italian normative study (see Cornoldi, Lucangeli, & Bellina, 2002), which was intended to examine the BA learning of children between ages 6 and 14 and involved more than 5,000 children. In the last part of this article, we will focus on some of the main results concerning the group of 3,595 children attending primary grades (Italian scuole elementari) which was part of the aforementioned larger group.

Italian Normative Study

The Italian normative study was developed in order to obtain normative data on the number and calculation abilities of Italian children. Boys and girls in Grades 1 through 5 (ages 6–11 years) were randomly selected in order to represent the Italian population (N = 3,595). Table 1 presents the composition of the sample. Different areas and different socioeconomic statuses were proportionally represented.

Method. The assessment procedure (Calculation Ability–MT Group; Cornoldi et al., 2002) devised for the study was a shortened version of the standardized Italian test called *Abilita di Calcoloz Calculation Ability* (Lucangeli, Tressoldi, & Fiore, 1998), which was inspired by McCloskey, Caramazza, and Basili's (1985) neuropsychological model. On the basis of patients with specific number and calculation disorders, this model assumes that different abilities can be dissociated. These abilities can be included within some main categories (i.e., comprehension of numbers, production of numbers, calculation, use of operation signs, use of procedures, numerical facts).

To test different aspects of calculation and to offer a quicker tool, we devised a rapid assessment procedure that includes a paper-and-pencil group testing session and an individual quick testing session. The following subtests were used in the group session:

- written calculation;
- size comparison (semantic numerical knowledge): six pairs of numbers (e.g., 1024 and 1402) are presented, and the child must decide which one is greater;
- word–number transcoding (lexical and syntactic numerical knowledge): the child must write the numbers corresponding to six series of lexical strings, reporting (in a random order) the numbers of units, teens, hundreds, and thousands; and
- number ordering (semantic and syntactic knowledge): the child must order, according to an increase or decrease in size, 10 series of four complex numbers.

In the individual session, the following tests were used, and the associated scores were obtained:

- mental calculation (accuracy and speed);
- written calculation (accuracy and speed);
- enumeration: forward from 1 to 20 (first grade), forward from 1 to 50 (second grade), backward from 100 to 50 (third, fourth, and fifth grade; accuracy and speed);
- number dictation; and
- numerical facts: basic knowledge of addition and subtraction with one-digit numbers, multiplication tables, and other facts is tested, and a response is considered correct only if it is given immediately and not as a result of a computation.

Results. The results were analysed separately for each subtest, each grade, and for two different periods of the year in each grade. For an overall view of their arithmetic performance, the children's scores at each subtest of a subgroup of children (n = 621) were transformed into z scores on the basis of the data for their particular grade.

A factor analysis was carried out on these children's z scores, using a promax rotation method for oblique factors, with a hypothesis of three partially related factors (numerical knowledge, written calculation accuracy, calculation speed). The analysis explained 54% of the variance.

In Table 2, saturations greater than .35 are reported. As can be seen, the analysis substantially confirmed the three-factor hypothesis. In fact, the first factor (automatization) saturated scores referred to mental calculation, where working memory resources, automaticity, and speed of processing may be critical (see Salthouse, 1994); knowledge of arithmetic facts, also based on automatization of knowledge; and the other specific speed measures. The second factor (numerical knowledge) saturated all the measures concerning numerical knowledge, and the third factor (written calculation) saturated written calculation.

Thus, the analysis showed that different main components can be found in calculation competence. As a consequence, in order to have more synthetic data, but also to give the possibility of separating the group and the individual sessions, two main scores were computed for the group testing, namely, written calculation and numerical knowledge (the latter obtained by summing the scores for all the other subtests), and two other main scores were computed for the individual testing, one for the overall number of errors and the other one for the overall response time for the three timed subtests (written and mental calculation, counting).

This gave us the opportunity to examine the general patterns of data; for example, we found that in the Italian population, no clear differences exist between boys and girls in general calculation abilities. Furthermore, different arithmetic competencies share many commonalities, as it appeared from the

TABLE 1
Composition of the Sample for the Italian Normative Study (Grades 1–5)

Grade	n	Boys	Girls	Mean age (mos.)
1	338	180	158	82.5
2	801	393	408	92.5
3	870	442	428	105.0
4	820	427	393	116.5
5	766	372	394	129.0

Note. N = 3,595.

TABLE 2
Promax Factor Analysis of the Performance of Italian Children on a Battery for the Assessment of Calculation Abilities

	Factor		
Subtest	**1**	**2**	**3**
Group testing			
Written calculation			.783
Size comparison		.530	
Word–number transcoding		.662	
Number ordering		.775	
Mental calculation			
Accuracy	.719		
Speed	.809		
Individual written calculation			
Accuracy			–.579
Speed	.625		
Enumeration			
Errors		–.522	
Speed	.769		
Number Dictation		–.623	
Numerical facts	.547		

Note. Negative values for some saturations are due to the fact that the corresponding measures yielded the number of errors rather than the number of correct responses.

significant correlations between them and was corroborated by a further study, which contrasted them with another type of learning (i.e., reading). In fact, with a group of 215 children from Grades 3, 4, and 5, we had the opportunity to contrast their reading and arithmetic achievements and to analyze their association with two basic intelligence factors as classically defined by Thurstone and Thurstone (1963; i.e., verbal and numerical). For this purpose we administered a standardized reading test (MT test; Cornoldi & Colpo, 1981) and the two primary abilities subtests originally designed by Thurstone and Thurstone in order to assess the children's verbal and numerical abilities, involving vocabulary knowledge and numerical reasoning respectively. Table 3 presents the factor analysis solution obtained for the main calculation and reading scores (varimax rotation, 63% of the variance explained) with saturations greater than .35. It is clear that when contrasted with another type of learning, calculation abilities appear to share many similarities with a numerical ability factor and are differentiated from a verbal skills factor.

Implications for Subtypes of ALD

Our data offer a further contribution to the effort of subtyping ALD (e.g., Rourke & Strang, 1983; Shafrir & Siegel, 1994). In fact, the differentiation between different components of learning not only concerned the analysis of individual differences within the overall population of children but found further support in the analysis of individual children with ALD.

In our view, this effort is important, as it indicates the lines for the differentiation of diagnosis and remediation procedures. From our studies, it appears that subtyping may be done at different levels of subtlety according to the articulation level possible in the analysis of single cases. We illustrate here a five-levels approach to subtyping that is mainly based on the performance on achievement tests. These levels form a diagnostic tree that can be used in the analysis of ALD (see Figure 1).

A first level concerns the general category of disability. Specific arithmetic learning disabilities (ALD) must be distinguished from arithmetic difficulties mainly due to other related factors, such as mental retardation, attention-deficit/hyperactivity disorder (ADHD), and so on.

A second level concerns the analysis of general learning areas. An ALD can be associated with other types of difficulties, for example, reading disabilities (Shafrir & Siegel, 1994), visuospatial (nonverbal) learning disabilities (Rourke & Strang, 1983), but can also be independent, as some children with ALD do not present reading difficulties (Shafrir & Siegel, 1994) and visuospatial disabilities do not overlap with arithmetic disabilities (Cornoldi, Rigoni, Tressoldi, & Vio, 1999). This partial independence corresponds with the independence of the underlying numerical, linguistic, and visuospatial forms of intelligence (see also Table 3).

A third level concerns differentiation among types of ALD. In particular, in the present article, we mention evidence in favor of the distinction between specific difficulties in basic arithmetic abilities (BA) involved in calculation and difficulties in APS (i.e., in the solution of arithmetic problems typically, but not necessarily, presented in a linguistic format). This differentiation is substantiated by a consistent body of evidence presenting cases of children who have problems in BA (sometimes defined as dyscalculia) but who can solve a reasonably high number of problems if they are not under calculation pressure. On the other hand, some children may have an average ability in calculation but meet a greater difficulty in solving problems (Jordan & Montani, 1997). This dissociation seems to a certain extent to mirror the dissociation between reading decoding and reading comprehension disabilities, as both cases involve a differentiation between basic abilities, which must be overlearned and automatized, and more complex abilities, which even when some routine procedures can be used (e.g., typical solution algorithms), still continue to require the use of controlled processes.

A fourth level is involved in the differentiation of different subtypes of disabilities in BA. According to the data of the Italian normative study (single cases are presented in Cornoldi et al., 2002) and to the factor analysis presented in Table 2, at this level three main types can be distinguished, depending on whether the disability mainly concerns numerical knowledge, the use of procedures in written calculation, or mental and automatized calculation. It must be noted that numerical knowledge is a prerequisite of the other two aspects, and some aspects of mental and automatized calculation are required in doing written calculation (in fact, the three aspects appear to be highly correlated; see Cornoldi et al., 2002). However, cases of dissociation can be found. For example, some children possess only an elementary numerical knowledge or a low level of automatization but nevertheless do not do worse than many of their classmates on written calculation. In contrast, other children may have good numerical knowledge but score low in calculation, either because they are accurate but dramatically slow or because they are fast but make a lot of errors.

A fifth level concerns a deeper analysis of the arithmetic difficulties that are met by the child. According to the assessment procedure described here, this differentiation could be done on the basis of the performance in the individually administered tasks. For example, a child may have difficulties in learning numerical facts that are more severe than in learning other aspects of calculation, including other aspects of the mental calculation/automatization component. This case was rather frequent in our normative sample. As we mentioned previously, the definition of the tasks for our assessment procedures originated from an analysis of different components of number knowledge and calculation in adult patients (McCloskey et al., 1985; see Level 5 in Figure 1). At this moment, we do not know to what extent a similar analysis can be extended to children. If this extension will not be sufficiently supported, a more qualitative analysis of performance can be used. In fact, as suggested by Wong (1996), a different approach can look more specifically at the most frequent types of errors. A good example of this approach can be represented by the identification of children who make a lot of syntactic errors in numbers including zero (for example, after the dictation of *one thousand and three*, to write 10003) but do not present a general difficulty in treating other syntactic aspects of numbers.

TABLE 3
Varimax Factor Analysis of the Performance of Italian Children Basic Numerical and Linguistic Abilities and Reading and Arithmetic Achievement

Subtest	Factor 1	Factor 2
Group numeracy		
Written calculation	.716	
Numerical knowledge	.552	
Individual numeracy		
Errors	–.638	
Speed	–.715	
Reading		
Speed		.864
Errors		.868
Intelligence		
Verbal		–.573
Numerical	.778	

Note. Negative values for some saturations are due to the fact that the corresponding measures yielded the number of errors rather than the number of correct responses.

Conclusions

In the present article, we have offered an overview of the most recent trends of research on mathematical learning in Italy. Although these trends partially reflect trends also present in other countries, they have some specific elements. In particular, the recently concluded Italian normative study on calculation abilities (which probably represents the largest effort ever made in Italy in this context) offers a general view of arithmetic learning and of types of arithmetic abilities and disabilities.

This view gave us the opportunity to define different levels of subtyping of ALD and to create a diagnostic tree. A five-level diagnostic tree might appear excessively complex and ambitious, but it is in our view more satisfactory than other approaches both at a theoretical and at a clinical–educational level.

At a theoretical level, a more subtle articulation of arithmetic difficulties better describes the complexity of arithmetic learning. At a clinical or educational level, this articulation may guide assessment and the identification of specific rehabilitative goals. For example, it is obvious that children who do poorly in arithmetic because they are inaccurate (an aspect frequently considered by teachers, at least in Italy) require a different type of intervention than children who are accurate but slow; the latter often receive less attention, as their difficulties are less evident, but they may have severe troubles in all the cases where there are time constraints or where rapid and

Level 1

Is it a specific learning disability?

Level 2

If yes, does the disability concern arithmetic or only other aspects of learning (reading, visuospatial learning, etc.)?

Level 3

If yes, does the arithmetic learning disability mainly concern calculation or problem solving?

Level 4

If it concerns calculation, does the disability mainly concern,

1. numerical knowledge?
2. accuracy of written calculation?
3. speed/automatization?

Level 5

Which specific aspect is mainly involved?

1. numerical knowledge: productive vs. receptive processes; lexical vs. semantic vs. syntactic knowledge
   - Size comparison
   - Word–number transcoding
   - Number ordering
   - Enumeration
   - Number dictation
2. accuracy of written operation: which operation in particular?
3. speed /automatization:
   - accuracy of mental calculation
   - speed of mental calculation
   - speed of written calculation
   - enumeration speed
   - numerical facts

FIGURE 1. Diagnostic tree for the evaluation of arithmetic learning disabilities.

automatized calculation is necessary in order to meet specific requests.

Furthermore, a diagnosis made following our diagnostic tree is not particularly complicated because it can be based on a deeper analysis of the assessment device presented here, whose administration requires approximately 20 minutes of group testing and 10 minutes of individual testing, and it can be supported a second time by further examination. The assessment should determine the degree of the child's difficulty in one of the three main arithmetic variables (Level 4 of the diagnostic tree in Figure 1) and then the contribution of each of the specific variables described at Level 5. For example, some children show specific speed–automatization deficits, whereas their accuracy is good. These cases seem more sensitive to treatment than children with the opposite pattern of performance (low accuracy, moderately high speed) as they are not affected by learned poor procedures and incorrect arithmetical facts. Treatment should build on the existing competencies and increase speed through practice and strategy training. Furthermore, as these difficulties do not always involve all the specific underlying variables, a deeper consideration should examine which variable is mainly affected. If, for example, mental calculation is involved, it is particularly evident that strategies play a critical role, as the child may either know a limited number of strategies or use them in a poor way. The intervention should then be based not only on repeated exercise in mental calculation, but also on the teaching of a variety of different strategies and on their flexible and efficient use in retrieval, selection, and implementation.

ABOUT THE AUTHORS

Cesare Cornoldi, *PhD, is full professor in general psychology at the University of Padova, Italy. His current interests concern memory, mental imagery, and learning disabilities. He is the president of the Italian Society for Learning Disabilities (AIRIPA).* ***Daniela Lucangeli,*** *PhD, is an associated professor in educational and developmental psychology at the University of Padova. Her current interests concern learning and educational psychology, with particular reference to mathematical learning. She is a member of many national and international research associations. Address: Daniela Lucangeli, Department of Developmental Psychology, University of Padova, Via Venezia, 8, 35131 Padova, Italy; e-mail: daniela.lucangeli@unipd.it*

REFERENCES

Arzarello, F. (1992). La ricerca in didattica della matematica [Research in mathematics education]. *L'Insegnamento della Matematica e delle Scienze Integrate, 15*, 345–356.

Arzarello, F. (1996). Tendenze attuali della ricerca italiana in didattica della matematica [Trends in Italian research on mathematical teaching]. *Notiziario UMI, 1–2*, 68–78.

Arzarello, F., & Bartolini-Bussi, M. (1998). Italian trends of research in mathematics education: A national case study in the international perspective. In J. Kilpatrick & A. Sierpinska (Eds.), *What is research in mathematics education and what are its results?* (pp. 243–262). Washington, DC: Kluwer.

Boscolo, P. (1997). *Psicologia dell'apprendimento scolastico* [Psychology of school learning]. Torino, Italy: Utet.

Castelnuovo, E. (1965). L'oggetto e l'azione nell'insegnamento della geometria intuitiva [Object and action in intuitive geometry teaching]. *Il Materiale per l'Insegnamento della Matematica 10,* 41–65.

Cornoldi, C., Lucangeli, D., & Bellina M. (2002). *AC-MT Test: Test per la valutazione delle difficoltà di calcolo* [The AC-MT arithmetic achievement test]. Trento, Italy: Erickson.

Cornoldi, C., & Colpo, G. (1981). *MT test.* Firenze, Italy: Organizzazioni Speciali.

Cornoldi, C., Rigoni, F., Tressoldi, P., & Vio, C. (1999). Imagery deficit in nonverbal learning disabilities. *Journal of Learning Disabilities, 32,* 48–57.

Jordan, N. C., & Montani, T. O. (1997). Cognitive arithmetic and problem solving: A comparison in children with specific and general mathematics difficulties. *Journal of Learning Disabilities, 30,* 629–639.

Lucangeli, D., & Cornoldi, C. (1997). Mathematics and metacognition: What is the nature of the relationship? *Mathematical Cognition, 2,* 121–139.

Lucangeli, D., Cornoldi, C., & Tellarini, M. (1998). Metacognitive difficulties in problem solving skills. *Advances in Learning Disabilities, 12,* 219–243.

Lucangeli, D., & Passolunghi, M. C., (1995), *Psicologia dell'apprendimento matematico* [Psychology of mathemetical learning]. Torino, Italy: Utet.

Lucangeli, D., Tressoldi, P., & Cendron, M. (1998). *SPM: Test di valutazione delle abilita' di soluzione dei problemi matematici* [The SPM test for math problem solving achievement]. Trento, Italy: Erickson.

Lucangeli, D., Tressoldi, P., & Cendron, M. (1999). Validation of a comprehensive model for mathematical word problem solving. *Contemporary Journal of Educational Psychology, 23,* 257–275.

Lucangeli, D., Tressoldi, P., & Fiore, C. (1998). *ABCA: Test di valutazione delle abilita' di calcolo aritmetico* [The ABCA arithmetic achievement test]. Trento, Italy: Erickson.

Malara, N. A., Menghini, M., & Reggiani, M. (1996). *Italian research in mathematics education: 1988–1995.* Rome: Litoflash.

McCloskey, M. (1992). Cognitive mechanisms in numerical processing: Evidence from acquired dyscalculia. *Cognition, 44,* 107–157.

McCloskey, M., Caramazza, A., & Basili, A. (1985). Cognitive mechanism in number processing and calculation: Evidence from dyscalculia. *Brain and Cognition, 4,* 171–96.

Passolunghi, M. C., Cornoldi, C., & De Liberto, S. (1999). Working memory and intrusions of irrelevant information in a group of specific poor problem solvers. *Memory and Cognition, 27,* 779–790.

Rourke, B. P., & Strang, J. D. (1983). Concept formation and nonverbal reasoning abilities of children who exhibit specific academic problems with arithmetic. *Journal of Clinical Child Psychology 12,* 33–39.

Salthouse, T. A. (1994). The aging of working memory. *Neuropsychology, 8,* 535–543.

Santagata, R., & Stigler J. W. (2000). *Teaching mathematics: Italian lessons from a cross-cultural perspective.* Unpublished manuscript.

Sempio, O. (1997). *Il bambino e la costruzione del numero* [Numerical knowledge in children]. Rome: La Nuova Italia Scientifica.

Shafrir, U., & Siegel, L. S. (1994). Subtypes of learning disabilities in adolescents and adults. *Journal of Learning Disabilities, 27,* 123–134.

Temple, C. M. (1991). Procedural dyscalculia and number fact dyscalculia: Double dissociation in developmental dyscalculia. *Cognitive Neuropsychology, 8,* 155–176.

Thurstone, T. G., & Thurstone, L. L. (1963). *Primary mental abilities for Grades 4–6: Examiner's Manual.* Chicago: Science Research Associates.

Unione Matemetica Italiana. (1980). *Syllabus di matimatica* [Mathematical syllabus]. Firenze, Italy: Author.

Wong, B. Y. L. (1996). *The ABCs of learning disabilities.* New York: Academic Press.